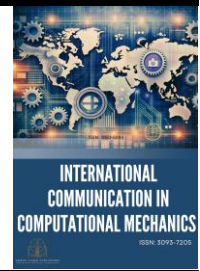




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# Analysis on the Configuration of Four Reaction Wheels on the Attitude Manoeuvring of the Satellite

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ARTICLE INFO	ABSTRACT
<p><b>Article history:</b>                      Received 7 November 2024                      Received in revised form 18 December 2024                      Accepted 7 December 2024                      Available online 15 March 2025</p> <p><b>Keywords:</b>                      Satellite attitude; attitude control;                      reaction wheels</p>	<p>Satellites require high precision and speed in their attitude control systems, making the selection of an optimal reaction wheel configuration critical. Given the variety of possible configurations, a comprehensive analysis is necessary to determine the maximum torque each configuration can generate. This thesis explores the optimization of satellite reaction wheel configurations through mathematical modelling and torque analysis. Various configurations were identified through a literature review, and the corresponding equations of motion were developed. The torque generated by the reaction wheels was evaluated at 0.1 Nm<sup>2</sup> along the z-axis. Three configurations were analysed, with angles <math>\alpha</math> and <math>\beta</math> representing the rotation and position of the reaction wheels. A custom code was used to calculate the torque for each configuration, and the results were visualized with convex hull plots. The analysis revealed significant differences in torque generation across configurations. The findings emphasize the importance of selecting the most suitable configuration based on specific mission requirements and wheel orientations for optimal satellite performance.</p>

## 1. Introduction

Achieving precise pointing control and rapid maneuvering capabilities is critical for meeting the stringent requirements of space missions [1]. Traditional approaches, such as the Quaternion Feedback attitude control system with augmented dynamics, address these challenges but often require complex parameter tuning, which can limit optimal performance [2]. Additionally, redundancy issues with satellite actuators complicate attitude control further, making system reliability a key concern [3]. The Attitude Determination and Control System (ADCS) plays a crucial role in ensuring a satellite meets its mission objectives by providing control signals to actuators like reaction wheels (RWs) [4].

Reaction wheels (RWs) are vital components in modern spacecraft attitude control systems, enabling precise orientation adjustments without the need for propellant-based thrusters [5]. Using the principle of conservation of angular momentum, reaction wheels adjust the satellite's orientation

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by varying the speed of internal rotating wheels. By spinning the wheels faster or slower, torque is generated, allowing the satellite to change its attitude without expelling mass or using fuel. This capability is particularly advantageous for long-duration missions, where maintaining orientation without fuel consumption is essential [6]. High-precision attitude control is crucial in missions such as astronomy or space-based telescopes, where even minute changes in orientation can impact data quality and scientific results [7].

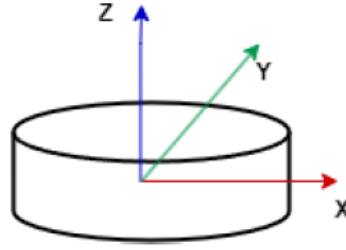
In the design of reaction wheel systems, the configuration of the wheels—typically arranged along the spacecraft's principal axes (x, y, and z)—is crucial for generating the necessary torques to control the satellite's orientation [8], [9]. While a common configuration involves three reaction wheels, one aligned along each axis, more complex configurations with additional wheels can improve performance and reliability [10]. Additional wheels enhance redundancy, ensuring the spacecraft can maintain control if one-wheel malfunctions, thereby improving fault tolerance [11], [12]. One key challenge with reaction wheel systems is wheel saturation, where a wheel reaches its maximum rotational speed and can no longer contribute to attitude control. To mitigate this, recent developments have focused on optimizing configurations to prevent saturation and enhance system robustness. Some systems incorporate four or more wheels to ensure continued control even if one wheel fails or experiences degradation [13].

The orientation of the wheels relative to the spacecraft's body axes is essential for enabling control in multiple directions, facilitating complex maneuvers required in missions like Earth observation, communication, and scientific experiments. Various reaction wheel configurations have been explored in the literature to optimize these factors [14]. Recent research has focused on improving the modeling and calculation of momentum envelopes, integrating advanced control strategies to maximize the usage of the available momentum before the wheels reach saturation [15]. Additionally, fault-tolerant and redundant systems have been developed, where multiple wheels work in conjunction to extend the momentum envelope and ensure system reliability in case of wheel failure or performance degradation [16]. Momentum envelope analysis, which evaluates the reaction wheels' ability to generate maximum torque in the desired directions, is used to assess and improve the effectiveness of different configurations [17]. These shows that the configuration of the RW plays an important role in the design of a satellite, understanding how the configuration affects the attitude maneuvering increases the success rate of the space mission. This paper aims to provide better insight the how the configuration of the RW affects the torque generation on three available designs.

## **2. Methodology**

### ***2.1 Modelling of the RW***

A typical RW generates torque along the z-axis when rotating about its axis, as shown in Figure 1. The direction of the generated torque depends on the direction of the wheel's rotation. For the purposes of this analysis, a clockwise rotation of the RW is considered to produce torque along the positive z-axis [18].



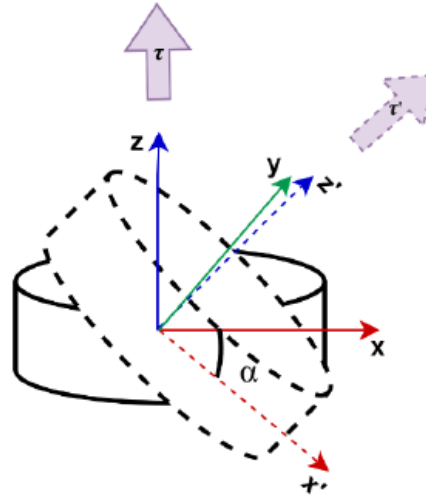
**Fig. 1.** Local axis of a RW

Thus, the torque produced by the RW in clockwise direction is given as Eq. (1), where the numbering for the RW,  $i=1, 2, 3$  and  $4$  [14]:

$$\tau_i = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} Nm \quad (1)$$

## 2.2 RW Rotation on Local Frame

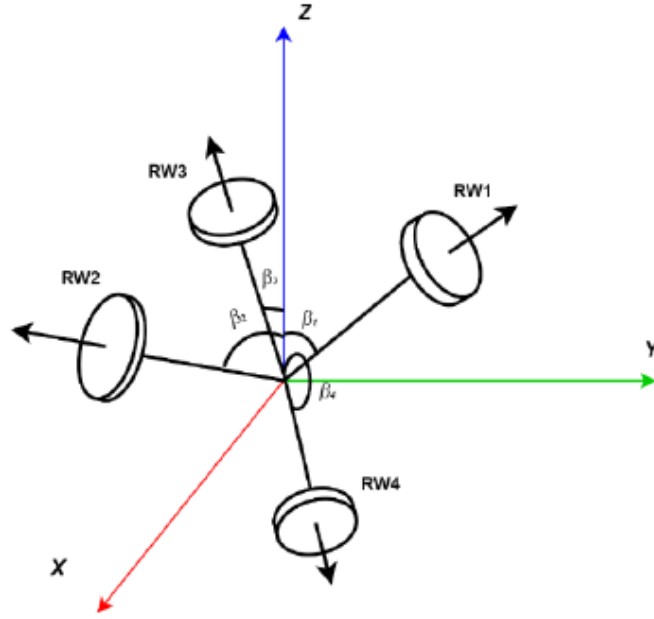
Figure 2 illustrates a RW rotated by an angle  $\alpha$  along its y-axis, relative to its original local reference frame (denoted by  $x, y, z$ ). After the rotation, the new local reference frame is labeled  $x', y', z'$ . As a result of this rotation, the direction of the applied torque  $\tau$  initially aligned with the Z-axis, also rotates by the same angle  $\alpha$ . The new torque direction after the frame rotation is denoted as  $\tau'$ . This demonstrates the effect of the frame rotation on the direction of the applied torque, highlighting how the orientation of the torque vector changes in response to the wheel's rotation.



**Fig. 2.** The rotation of RW by  $\alpha$  degrees along the y-axis

## 2.3 RW Rotation on Global Frame

Figure 3 shows the positions of all four RWs in the global reference frame. The angle  $\beta_1$  represents the rotation of the global reference frame along the z-axis for RW1. Similarly,  $\beta_2$ ,  $\beta_3$ , and  $\beta_4$  represent the rotations of the reference frame for RW2, RW3, and RW4, respectively [19].



**Fig. 3.** The rotation of RWs by  $\beta$  degrees from reference to global reference frame

#### 2.4 Equation of Motion of the RWs

The RWs undergo two rotations, represented by angles  $\alpha$  and  $\beta$ . Therefore, the model must include two transformation matrices. The transformation matrix  $R$ , which describes the rotation, is shown in Eq. (2), where  $i$  represents the index of the RW.

$$R_i = \begin{bmatrix} \cos \beta_i \cos \alpha_i & -\sin \beta_i & \cos \beta_i \sin \alpha_i \\ \sin \beta_i \cos \alpha_i & \cos \beta_i & \sin \beta_i \sin \alpha_i \\ -\sin \alpha_i & 0 & \cos \alpha_i \end{bmatrix} \quad (2)$$

Thus, the torque generated by the RW  $i$ , with rotational angles  $\alpha_i$  and  $\beta_i$ , is obtained by multiplying the transformation matrix by the torque matrix from Equation 1. This results in the torque model of the RWs, which is expressed in Eq. (3).

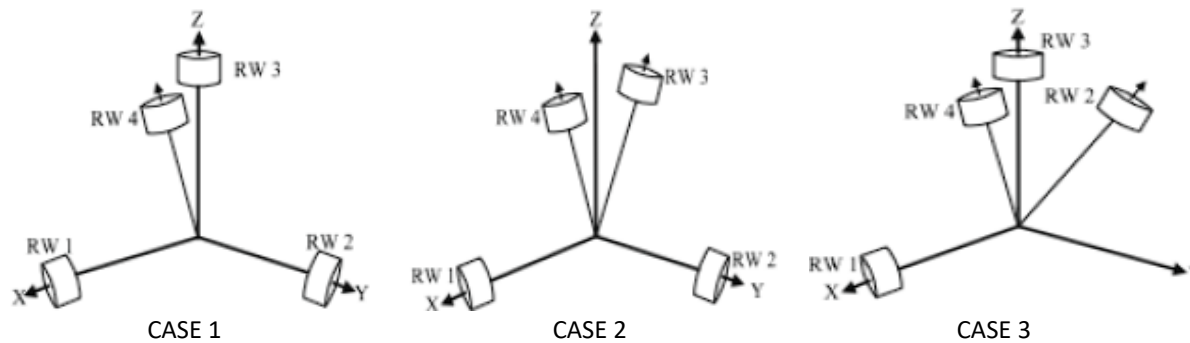
$$\tau_{i,rotate} = \begin{bmatrix} 0.1 \cos \beta_i \sin \alpha_i \\ 0.1 \sin \beta_i \sin \alpha_i \\ \cos \alpha_i \end{bmatrix} Nm \quad (3)$$

The total torque generated by all the RW is obtain as in Eq. (4).

$$\sum_{i=1}^4 \tau_{i,rotate} = \sum_{i=1}^4 \begin{bmatrix} 0.1 \cos \beta_i \sin \alpha_i \\ 0.1 \sin \beta_i \sin \alpha_i \\ \cos \alpha_i \end{bmatrix} Nm \quad (4)$$

## 2.5 Configuration of the RW

The configuration for the RWs used for this paper is as shown in Figure 4.



**Fig. 4.** Three cases of RW configurations [14]

The  $\alpha_i$  and  $\beta_i$  rotation of the RW are shown in table 1

**Table 1**

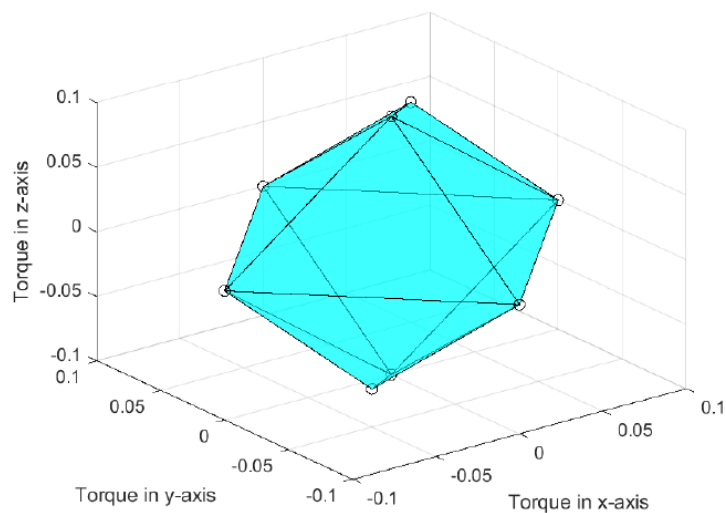
Angle of rotation of  $\alpha$  and  $\beta$

	CASE 1		CASE 2		CASE 3	
	$\alpha_i$	$\beta_i$	$\alpha_i$	$\beta_i$	$\alpha_i$	$\beta_i$
RW1, $i=1$	90°	0°	90°	0°	90°	0°
RW2, $i=2$	90°	90°	90°	90°	-45°	45°
RW3, $i=3$	0°	0°	-45°	-45°	0°	0°
RW4, $i=4$	45°	45°	45°	45°	45°	45°

Calculate the Eq. (4) is done using MATLAB software where the momentum envelope is obtained using the Convhull function to show the extreme of torque generated throughout 360 degrees using the angles from Table 1. The analysis is done based on the assumption that there is no loss of momentum during the operation of the RW.

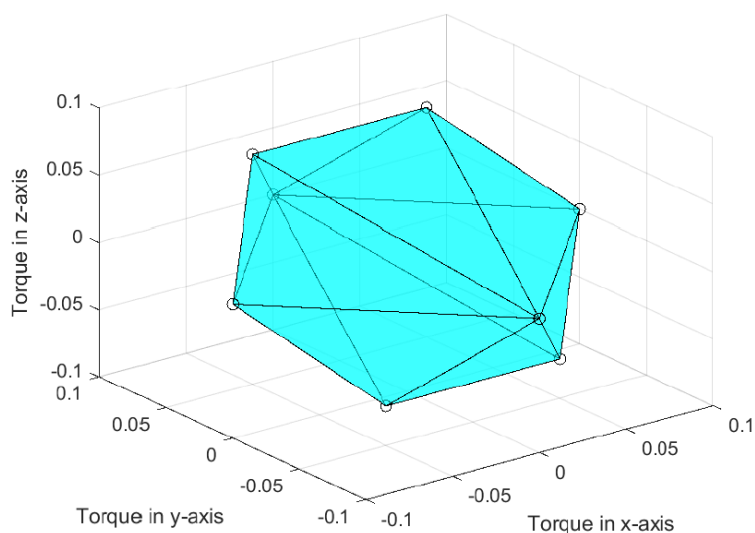
## 3. Results

To assess the performance of the RW configuration, the momentum envelope is calculated to illustrate the extent of the momentum generated by all the reaction wheels in a 3D sphere [20]. Momentum envelopes of case 1, case 2 and case 3 are shown in Figure 5-7. As shown in Figure 5, the RW configuration in Case 1 produces the highest torque on the z-axis, with a value of  $\pm 0.1707$  Nm. The shape of the momentum envelope indicates that the torque generation is almost uniformly distributed across the full 360 degrees. The Case 1 configuration provides a well-distributed torque, making it suitable for satellite systems that require rotation across all 360 degrees.



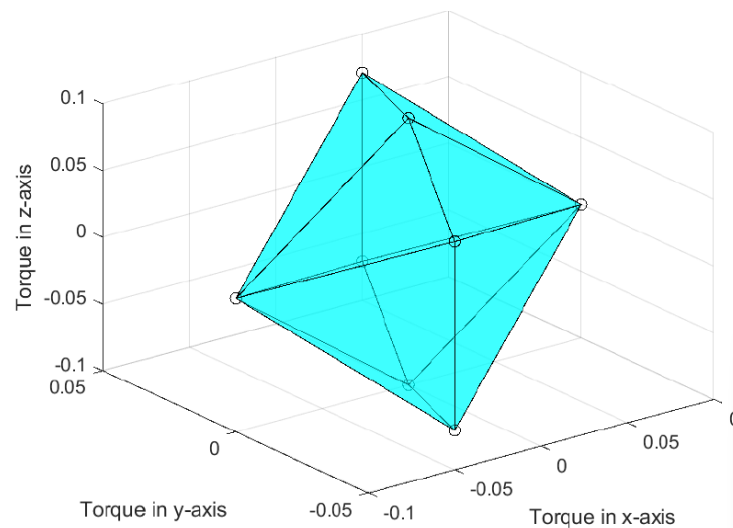
**Fig. 5.** Momentum envelope for Case 1

In the case of Configuration 2, the torque performance is shown in Figure 6. The RWs produce a maximum torque of  $\pm 0.2$  Nm along the y-axis, while the minimum torque is  $\pm 0.1$  Nm along the x-axis. This configuration is ideal for satellites that require greater rotational control along the y-axis, with less emphasis on x-axis rotation.



**Fig. 6.** Momentum envelope for Case 2

Figure 7 shows the momentum envelope for the Case 3 configuration. This configuration produces a high torque value of  $\pm 0.2414$  Nm along the z-axis, while the torque along the y-axis is nearly zero. Therefore, this configuration is well-suited for satellite missions that require significant maneuvering along the z-axis, with minimal or no rotation around the y-axis.



**Fig. 7.** Momentum envelope for Case 3

## 4. Conclusions

In conclusion, the study demonstrates that the configuration of the reaction wheels (RWs) has a significant impact on the performance of the satellite's attitude control system. Selecting the appropriate configuration when designing the satellite's attitude maneuvering system is crucial. The right configuration enhances the likelihood of mission success while minimizing potential risks.

## Acknowledgement

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