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On The Third Logarithmic Coefficient Estimates for Certain Generalized Subclass of Close-to-Convex Functions

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ABSTRACT

In this paper, we introduce a new class of generalized close-to-convex functions, which satisfies the following condition, $Re\left\{e^{i\lambda}\frac{zf'(z)}{g(z)}\right\} > \delta$, where $g(z) = \frac{z}{(1-z^2)^{\beta}}$, $0 \leq \beta \leq 2$, $|\lambda| < \frac{\pi}{2}$, $cos\,\lambda > \delta$, and $z \in E = \{z \in \mathbb{C}: |z| < 1\}$. The third logarithmic coefficient estimates are considered, and we obtain sharp inequality for the class. Our results extend and unify previous work on starlike, convex, and close-to-convex functions under representation theorem.

1. Introduction

The phrase of univalent functions has a considerably broader meaning, referring to regular (holomorphic) or meromorphic functions that determine a one-to-one mapping. Although univalent functions can be defined in a variety of domains, including on a Riemann surface, most emphasis is focused on a few specific classes. Let S be subclass of analytic function A, normalized by $f(0) = f'(0) - 1 = 0$, with $|z| < 1$ defined as the unit disk, E, such as $z \in E$. If the function $f \in A$, then $f(z)$ has the following Taylor series form,

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \quad (1)$$

The Koebe function is known as an extremal function and this function also plays an important role in class S for maximizing $|a_n|$ for every n [1]. This function can be represented as follows:

$$K(z) = \frac{z}{(1-z)^2} = z + 2z^2 + 3z^3 + \cdots + nz^n = \sum_{n=2}^{\infty} nz^n. \quad (2)$$

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The class of functions with positive real parts can be denoted as P. The function with a positive real part, $p(z) \in P$, can be represented in the following form, gave

$$p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n = 1 + p_1 z + p_2 z^2 + \cdots + p_n z^n, \tag{3}$$

where $z \in E$, and $Re\big(p(z)\big) > 0$. The Mobius function can be written as,

$$L_0(z) = \frac{1+z}{1-z} = 1 + 2z + 2z^2 + 2z^3 + \cdots = 1 + 2\sum_{n=1}^{\infty} z^n, \tag{4}$$

and it plays an important role in P like the role of the Koebe functions in S [2].

There are three main subclasses in S such as starlike functions, convex functions and close-to-convex functions. The function f is a starlike if and only if

$$Re\left(\frac{zf'(z)}{f(z)}\right) > 0, \tag{5}$$

where $f \in S$ and $z \in E$. The class of starlike function is denoted by S^*.Next, the class is when function f become convex if and only if,

$$Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > 0, \tag{6}$$

where $f \in S$ and $z \in E$. This class is denoted by K. Lastly is the class of close-to-convex functions that have been stated by [3] that if function $f \in S$ and there exists a real number λ where $-\frac{\pi}{2} < \lambda < \frac{\pi}{2}$ and a function $g(z)$ is convex which satisfy these conditions,

$$Re \ \left\{e^{i\lambda} \frac{f'(z)}{g'(z)}\right\} > 0, \tag{7}$$

where $z \in E$, then the function f is a close-to-convex function. Alexander in 1916 showed the connection between starlike and convex functions if function $h(z) \in S^*$ then $h(z) = zg'(z)$ where $g(z) \in K$ [1]. Hence, the condition in Eq. (7) also can be written as

$$Re \ \left\{\frac{zf'(z)}{h(z)}\right\} > 0, \tag{8}$$

where $z \in E$. All starlike and convex functions are close-to-convex functions. It can be summarized by proper inclusion $K \subseteq S^* \subseteq C \subseteq S$. The class of close-to-convex functions is denoted by C.

The logarithmic coefficients for the function $f \in S$ are defined as,

$$log \frac{f(z)}{z} = 2 \ \sum_{n=1}^{\infty} \gamma_n z^n, \tag{9}$$

and the logarithmic coefficients are denoted by γ_n. The exact upper bounds for logarithmic coefficients are the very least. Milin's conjecture showed that for functions $f \in S$, and $n \geq 2$,

$$\sum_{m=1}^{n}\sum_{k=1}^{m}\left(k|\gamma_k|^2-\frac{1}{k}\right)\leq 0, \tag{10}$$

and the inequality in the form of Eq. (10) is not difficult to see that it is implies the Biebarbach conjecture. It was a proof of the inequality in the form of Eq. (10) that DeBranges established to prove conjecture [4]. In addition, the result of an average sense in [2] and [5] gives more attention than the exact upper bound for logarithmic coefficient $|\gamma_n|$.

Since the Koebe function plays an important role in the extremal function for a variety of extremal problems in class S , and it is expected that the upper bounds $|\gamma_n|\leq\frac{1}{n}$ hold for the function in S. Generally, the upper bound for $|\gamma_n|$ is not true, even in order of magnitude [4]. Several studies on the coefficient estimate for logarithmic coefficients for their own classes that have been established, for example, the results in [4], [6-12]. In addition, some research on the coefficients estimates problems in the Hankel determinants whose entries are the logarithmic coefficients has been established. Examples of the results can be obtained in [13-16].

From there, the authors were inspired to obtain the third logarithmic coefficient for the class of close-to-convex functions, where the function $g(z)$ is in the generalized form. The authors defined the class $f\in Q(2,\beta,\lambda,\delta)$, which is a class of the close-to-convex function that satisfies the following condition,

$$Re\left\{e^{i\lambda}\frac{zf'(z)}{g(z)}\right\}>\delta, \tag{11}$$

where $g(z)=\frac{z}{(1-z^2)^\beta}$, $0\leq\beta\leq 2$, $|\lambda|<\frac{\pi}{2}$, $cos\,\lambda>\delta$, and $z\in E$. The following section will be the preliminary results which includes the established lemmas that will be used to obtain the coefficient estimates problem.

2. Preliminary Results

In this section, we give two lemmas to prove our main result.

Lemma 1 (Libera and Zlotkiewicz [17-18])

Let $p\in P$ be in the form (3), then

$$c_1=2\zeta_1 \tag{12}$$

$$2c_2=2\zeta_1^2+2(1-|\zeta_1|^2)\zeta_2 \tag{13}$$

$$c_3=2\zeta_1^3+4(1-|\zeta_1|^2)\zeta_1\zeta_2-2(1-|\zeta_1|^2)\bar{\zeta}_1\zeta_2^2+2(1-|\zeta_1|^2)(1-|\zeta_2|^2)\zeta_3 \tag{14}$$

for some $\zeta_i\in\bar{E}(i\in 1,2,3)$, where $E:=\{z\in C:|z|<1\}$,$\bar{E}:=\{z\in C:|z|\leq 1\}$, and $\mathbb{T}:=\delta E$.

For $\zeta_1\in\mathbb{T}$, there is a unique function $p\in P$ with c_1 as in Eq. (12), namely,

$$p(z) = \frac{1+\zeta_1 z}{1-\zeta_2 z}, \tag{15}$$

where $z \in E$.

For $\zeta_1 \in E$, and $\zeta_2 \in \mathbb{T}$, there is a unique function $p \in P$ with c_1 and c_2 as in the Eq. (12)and Eq. (1), namely,

$$p(z) = \frac{1+(\bar{\zeta}_1\zeta_2+\zeta_1)z+\zeta_2 z^2}{1+(\bar{\zeta}_1\zeta_2-\zeta_1)z-\zeta_2 z^2}, \tag{16}$$

where $z \in E$.

For $\zeta_1, \zeta_2 \in E$, and $\zeta_3 \in \mathbb{T}$, there is a unique function $p \in P$ with c_1, c_2, and c_3 as in the Eq.(12) to Eq. (14), namely,

$$p(z) = \frac{1+(\bar{\zeta}_2\zeta_3+\bar{\zeta}_1\zeta_2+\zeta_1)z+(\bar{\zeta}_1\zeta_3+\zeta_1\bar{\zeta}_2\zeta_3+\zeta_2)z^2+\zeta_3 z^3}{1+(\bar{\zeta}_2\zeta_3+\bar{\zeta}_1\zeta_2-\zeta_1)z+(\bar{\zeta}_1\zeta_3+\zeta_1\bar{\zeta}_2\zeta_3-\zeta_2)z^2-\zeta_3 z^3}, \tag{17}$$

where $z \in E$.

Lemma 2 (Choi et al. [19])

This lemma is a special case of the more general results due to [19]. Defined as follows,

$$Y(A,B,C) := max\{|A+Bz+Cz^2|+1-|z|^2 \ : \ z \in \bar{E}\},$$

where A, B, and $C \in R$.

I. Case if $AC \geq 0$, then

$$Y(A,B,C) = \begin{cases} |A|+|B|+|C|, & |B| \geq 2(1-|C|), \\ 1+|A|+\dfrac{B^2}{4(1-|C|)}, & |B| < 2(1-|C|). \end{cases}$$

II. Case if $AC < 0$, then

$$Y(A,B,C) = \begin{cases} 1-|A|+\dfrac{B^2}{4(1-|C|)}, & -4AC\left(\dfrac{1}{C^2}-1\right) \leq B^2 \wedge |B| < 2(1-|C|), \\ 1+|A|+\dfrac{B^2}{4(1-|C|)}, & B^2 < min\left\{4(1+|C|)^2, -4AC\left(\dfrac{1}{C^2}-1\right)\right\} \\ R(A,B,C) & otherwise, \end{cases}$$

where

$$R(A,B,C):=\begin{cases} |A|+|B|-|C|, & |C|(|B|+4|A|)\leq |AB|, \\ -|A|+|B|+|C|, & |AB|\leq |C|(|B|-4|A|), \\ (|C|+|A|)\sqrt{1-\dfrac{B^2}{4AC}} & otherwise. \end{cases}$$

The next section will be the proof of Theorem 1 for the third logarithmic coefficients of the class close-to-convex functions.

3. Main Results

The investigation on the third logarithmic coefficient for the generalized class of close-to-convex functions attracted interest. The following theorem will show the working proof on obtaining the third logarithmic coefficient for the generalized class. Assuming that, $\lambda = 0$ and $\delta = 0$ for the function $f \in Q(2, \beta, \lambda, \delta)$ without the loss of generality in finding the third logarithmic coefficient. This analytical method will refer to the established lemmas of past research.

Theorem 1 *If* $f \in Q(2, \beta, 0,0)$ *is of the form of Eq. (1), then*

$$|\gamma_3| \leq \frac{1}{36}\left(\frac{(32-9\beta)\sqrt{64-18\beta}}{27} + 4\beta - \frac{13}{27}\right).$$

This inequality is sharp.

Proof. Let $f \in Q(2, \beta, 0,0)$ be of the form in *Eq. (1),* then there exists function $p \in P$ of the form in *Eq. (3),* such that

$$(1 - z^2)^\beta f'(z) = p(z) \tag{18}$$

where $z \in E$. Noted that, the coefficient of $f(z)$ by differentiating the *Eq. (18)*, yields

$$a_2 = \frac{1}{2}c_1 \tag{19}$$

$$a_3 = \frac{1}{3}(c_2 + \beta) \tag{20}$$

and

$$a_4 = \frac{1}{4}(c_3 + c_1\beta). \tag{21}$$

Also, the logarithmic coefficient gives

$$\gamma_1 = \frac{1}{2}a_2 \tag{22}$$

$$\gamma_2 = \frac{1}{2}\left(a_3 - \frac{1}{2}a_2^2\right) \tag{23}$$

and

$$\gamma_3 = \frac{1}{2}\left(a_4 - a_2 a_3 + \frac{1}{3}a_2^3\right). \tag{24}$$

Then, by substituting the coefficient $f(z)$ in Eq. (19) to Eq. (21), into the logarithmic coefficients in Eq. (22) to Eq. (24), yielding

$$\gamma_1 = \frac{1}{2}\left(\frac{1}{2}c_1\right) \tag{25}$$

$$\gamma_2 = \frac{1}{2}\left(\frac{1}{3}(c_2 + \beta) - \frac{1}{2}\left(\frac{1}{2}c_1\right)^2\right) \tag{26}$$

and

$$\gamma_3 = \frac{1}{2}\left(\frac{1}{4}(c_3 + c_1\beta) - \left(\frac{1}{2}c_1\right)\left(\frac{1}{3}(c_2 + \beta)\right) + \frac{1}{3}\left(\frac{1}{2}c_1\right)^3\right) \tag{27}$$

Next, by applying Lemma 1, for the logarithmic coefficient in the Equation 25 to Equation 27, will get the following equations,

$$\gamma_1 = \frac{\zeta_1}{2} \tag{28}$$

$$\gamma_2 = \frac{1}{2}\left(\frac{\zeta_1^2}{6} + \frac{2(1-|\zeta_1|^2)\zeta_2}{3} + \frac{\beta}{3}\right) \tag{29}$$

and

$$\gamma_3 = \frac{1}{48}[4\zeta_1^3 + 8(1-|\zeta_1|^2)\zeta_1\zeta_2 - 12(1-|\zeta_1|^2)\bar{\zeta}_1\zeta_2^2 + 12(1-|\zeta_1|^2)(1-|\zeta_2|^2)\zeta_3 + 4\zeta_1\beta]. \tag{30}$$

a. Assuming that $\zeta_1 = 1$, then by the Equation 30, give

$$|\gamma_3| \le \frac{(1+1\beta)}{12}.$$

b. Assuming that $\zeta_1 = 0$, then inequality will show,

$$|\gamma_3| \le \frac{1}{4}(1-|\zeta_2|^2)$$
$$|\gamma_3| \le \frac{1}{4}.$$

c. Assuming that $\zeta_1 \in (0,1)$. Since the inequality $|\zeta_3| \leq 1$, Eq. (30), will return as,

$$48\gamma_3 = 12(1-|\zeta_1|^2)\left[\frac{\zeta_1(\zeta_1^2+\beta)}{3(1-|\zeta_1|^2)} + \frac{2}{3}\zeta_1\zeta_2 - \bar{\zeta}_1\zeta_2^2 + 1 - |\zeta_2|^2\right]$$

The equation above can also be written as follows,

$$48\gamma_3 = 12(1-|\zeta_1|^2)[A_1 + B_1\zeta_2 + C_1\zeta_2^2 + 1 - |\zeta_2|^2] \tag{31}$$

where

$A = \frac{\zeta_1(\zeta_1^2+\beta)}{3(1-|\zeta_1|^2)}$, $B = \frac{2}{3}\zeta_1$, and $C = -\zeta_1$.

As mentioned in Lemma 2 for the case of the inequality$AC < 0$ for $\zeta_1 \in (1,0)$. Evaluate the inequality for $-4AC(C^{-2}-1) \leq B^2$ and $|B| < 2(1-|C|)$ as follow

$-B^2 - 4AC(C^{-2}-1) \leq 0,$

will gives,

$\frac{4}{9}(11\zeta_1^2 + 12\beta) \leq 0,$

which is false for the $\zeta_1 \in (1,0)$. Then for the inequality $|B| < 2(1-|C|)$ for $\zeta_1 \in (1,0)$, gives

$|B| - 2(1-|C|) < 0,$

yields,

$\frac{2}{3}|\zeta_1| - 2 + 2|\zeta_1| < 0,$

and

$\frac{8}{3}|\zeta_1| - 2 < 0,$

which is true for the$\zeta_1 \in (1,0)$. Since only the inequality $|B| < 2(1-|C|)$ satisfies $\zeta_1 \in (1,0)$ and the inequality does not satisfy $\zeta_1 \in (1,0)$, then the following condition does not satisfy,

$-4AC(C^{-2}-1) \leq B^2 \wedge |B| < 2(1-|C|)$

The investigation for another inequality condition on the Lemma 2 which is

$B^2 < min\{4(1+|C|)^2, -4AC(C^{-2}-1)\}.$

From that inequality,

$B^2 - 4(1 + |C|)^2 < 0,$

will give,

$\frac{32}{9}\zeta_1^2 - 8|\zeta_1| - 4 < 0.$

Examining another inequality which is $B^2 + 4AC(C^{-2} - 1) < 0$, gives,

$\frac{4}{9}\zeta_1^2 + 4(-1)\left[\frac{(\zeta_1^2+\beta)}{3(1-|\zeta_1|^2)}\right](1 - \zeta_1^2) < 0,$

and

$-\frac{8}{9}\zeta_1^2 - \frac{4}{3}\beta < 0.$

The condition of $B^2 < min\{4(1 + |C|)^2, -4AC(C^{-2} - 1)\}$ is satisfied. Therefore, by Lemma 2, and from Eq. (31), yielding,

$\gamma_3 = \frac{1}{4}(1 - |\zeta_1|^2)[A + B\zeta_2 + C\zeta_2^2 + 1 - |\zeta_2|^2],$

which gives

$$\gamma_3 = \frac{1}{4}(1 - |\zeta_1|^2)\left[1 + \frac{|\zeta_1|(\zeta_1^2 + \beta)}{3(1 - |\zeta_1|^2)} + \frac{\zeta_1^2}{9(1 + |\zeta_1|)}\right]$$

and

$\gamma_3 = \frac{1}{36}[9 + 3|\zeta_1|\beta - 8|\zeta_1|^2 + 2|\zeta_1|^3].$

From the above equation, it shows that

$36\gamma_3 = 9 + 3|\zeta_1|\beta - 8|\zeta_1|^2 + 2|\zeta_1|^3 \Rightarrow \vartheta(\zeta_1),$

and differentiate the function $\vartheta(\zeta_1)$ with respect to ζ_1 for obtaining the solution and gives,

$\frac{d\vartheta(\zeta_1)}{d\zeta_1} = 0,$

and

$3\beta - 16|\zeta_1| + 6|\zeta_1|^2 = 0.$

There are two roots for the function $\vartheta(\zeta_1)$, so it yields,

$\zeta_1 = \frac{4}{3} + \frac{\sqrt{64-18\beta}}{6}$, and $\zeta_1 = \frac{4}{3} - \frac{\sqrt{64-18\beta}}{6}$.

The root $\zeta_1 = \frac{4}{3} - \frac{\sqrt{64-18\beta}}{6}$ for the function $\vartheta(\zeta_1)$ will be selected because it satisfies $\zeta_1 \in (1,0)$.

Substitute the root in the function $\vartheta(\zeta_1)$ and gives,

$\vartheta\left(\frac{4}{3} - \frac{\sqrt{64-18\beta}}{6}\right) = \frac{(32-9\beta)\sqrt{64-18\beta}}{27} + 4\beta - \frac{13}{27}$.

Verifying that,

$\vartheta(\zeta_1) \leq \vartheta\left(\frac{4}{3} - \frac{\sqrt{64-18\beta}}{6}\right)$ for $\zeta_1 \in (1,0)$.

By tracing back the working of proof, it seems that the equality in the Theorem 1 holds when it follows the following condition,

$$\zeta_1 = \frac{4}{3} - \frac{\sqrt{64-18\beta}}{6} \tag{32}$$

$$\zeta_3 = 1 \tag{33}$$

and

$$|A + B\zeta_2 + C\zeta_2^2| + 1 - |\zeta_2|^2 = 1 + |A| + \frac{B^2}{4(1+|C|)}, \tag{34}$$

where

$A = \frac{(-8+\sqrt{64-18\beta})(-64-9\beta+8\sqrt{64-18\beta})}{-828+162\beta+144\sqrt{64-18\beta}}$,

$B = \frac{8}{9} - \frac{\sqrt{64-18\beta}}{9}$

and

$C = -\frac{4}{3} + \frac{\sqrt{64-18\beta}}{6}$.

Certainly, one of the solutions of Eq. (34) can be checked as follows:

$$\begin{aligned}\zeta_2 = \frac{1}{w_1}\{&-3240\beta^2 - 6(\sqrt{3})[-(\beta-1)(25515\beta^3 w_2 + 6561\beta^4 \\ &-897966\beta^2 w_2 - 829602\beta^3 + 6905412\beta w_2 + 13789116\beta^2 \\ &-14641000 w_2 - 71734104\beta + 117128000)]^{\frac{1}{2}} \\ &+38304\beta - 96800 + (81\beta^2 - 3132\beta + 12100)(w_2)\}\end{aligned} \tag{35}$$

where

$$w_1 = (243\beta^2 - 11988\beta + 49548)(\sqrt{64 - 18\beta}) - 11178\beta^2 + 150552\beta - 402216$$

and

$$w_2 = \sqrt{64 - 18\beta}$$

By substituting the Eq. (32), Eq. (33) and Eq. (35) into the Eq. (30), and further simplify it into the following inequality

$$36|\gamma_3| \leq \vartheta\left(\frac{4}{3} - \frac{\sqrt{64-18\beta}}{6}\right),$$

and gives

$$|\gamma_3| \leq \frac{1}{36}\left(\frac{(32-9\beta)\sqrt{64-18\beta}}{27} + 4\beta - \frac{13}{27}\right).$$

This ends the proof of the theorem. By letting the parameter $\beta = 1$ in Theorem 1, it will have the following Corollary 1. The result obtained can be reduced to the result in [9].

Corollary 1. *If* $f \in Q(2,1,0,0)$ *is of the form of Eq.* (*1*)*, then*

$$|\gamma_3| \leq \frac{1}{972}\left(95 + 23\sqrt{46}\right) \approx 0.258.$$

The inequality is sharp with the extremal function,

$$f(z) = \int_0^z \frac{p(t)}{1 - t^2}\,dt, \quad z \in E,$$

where

$$p(z) = \frac{(1+z)\left(9 + \left(7 - 2\sqrt{46}\right)z + 9z^2\right)}{(1-z)\left(9 + \left(1 + \sqrt{46}\right)z + 9z^2\right)}, \quad z \in E.$$

4. Conclusion

In this paper, we introduce a new function class $Q(2, \beta, \lambda, \delta)$, which is a generalized close-to-convex function. We solve for the third logarithmic coefficient estimates and obtain a sharp inequality for $Q(2, \beta, 0,0)$. The findings of the study can be extended to other forms of coefficient estimate, such as high-order Hankel and Toeplitz Determinant estimates. The theoretical results obtained in this study, particularly the sharp estimates for the third logarithmic coefficient in generalized close-to-convex functions, have meaningful implications in applied fields. These functions play a central role in conformal mapping, which is widely used in fluid dynamics, electromagnetic field modeling, and heat transfer analysis. Accurate coefficient bounds enhance the precision of such mappings, especially in complex geometries. Moreover, the findings can contribute to advancements in signal processing, computer graphics, and image transformation techniques, where analytic functions are used to ensure smooth and distortion-minimized transformations. The mathematical rigor established here also supports further development in symbolic computation tools and cryptographic systems that rely on complex function theory. Thus, while rooted in pure mathematics, the results offer foundational insights with potential applications across engineering, physics, and computational sciences.

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