KARYA ILHAM PUBLISHING
CONNECTING MINDS, BRIDGING IDEAS, INSPIRING READERS

Journal of Advanced Research in Social and Behavioural Sciences

Journal homepage:
https://karyailham.com.my/index.php/jarsbs/index
ISSN: 2462-1951

Making Sense of Calculus: Unpacking the Conceptual Knowledge of Instantaneous Rate of Change Among Malaysian Pre-Service Mathematics Teachers

Nurul Atiqah Talib[1,*], Suzieleez Syrene Abdul Rahim[2], Hutkemri Zulnaidi[2]

[1] School of Education, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia
[2] Department of Mathematics and Science Education, Faculty of Education, Universiti Malaya, Kuala Lumpur, Malaysia

ARTICLE INFO

Article history:
Received 31 December 2025
Received in revised form 23 February 2026
Accepted 30 April 2026
Available online 18 May 2026

ABSTRACT

While derivatives are often introduced through symbolic manipulation and graphical interpretations, their function as tools for modeling real-world change is rarely emphasized in secondary mathematics. A significant body of research has identified a persistent disconnect between students' procedural fluency and their conceptual knowledge of rate of change. This study investigates how Malaysian pre-service mathematics teachers conceptualize the derivative beyond its procedural definition, focusing specifically on how they interpret the concept of instantaneous rate of change across different forms: equations, tables, graphs, and symbols. Using a qualitative case study approach, three mathematics majors participated in task-based interviews that included interpretive questions. Data from interviews and written responses were analyzed through directed content analysis. Findings reveal that although participants could define derivatives as a "rate of change" or "slope," none could explain the underlying principles, such as the relationship between $\frac{dy}{dx}$ and $\frac{\delta y}{\delta x}$ or interpret how derivatives describe change in various representations. Their limited knowledge was closely tied to static procedures, lacking the flexibility needed for pedagogical application. This highlights an urgent need to shift teacher education from symbolic fluency towards conceptual adaptability, enabling future educators to teach derivatives as powerful tools for reasoning about change in diverse real-world settings.

Keywords:
Calculus; derivatives; pre-service mathematics teacher; rate of change; subject matter knowledge

1. Introduction

Derivatives in calculus are essential and contribute significantly to the world as they help describe relationships between changing quantities [22]. Since the world is inherently dynamic and constantly changing, such changes can be understood through rate of change. Some may occur dynamically, while others follow cyclic patterns [17]. Derivatives help us understand changing behaviors and their

* *Corresponding author.*
E-mail address: nurul.atiqah.talib@uum.edu.my

https://doi.org/10.37934/arsbs.43.1.193204

complex characteristics. By leveraging this advantage, many phenomena can be predicted, generating valuable data that benefits human life. There are fields that require better risk calculations [7], such as epidemiology, which involves the scientific study of how diseases spread and are controlled, and how populations change over time. Weather forecasting also considers many variables, such as changes in wind speed, temperature, and moisture levels, while manufacturing corporations require operational research analysts to observe changes in production costs to balance the company's finances by increasing profits and improving operational efficiency [17].Thus, the notion of the rate of change in calculus helps people understand upcoming possibilities and risks, enabling better decision-making and logical actions [22].

However, many students are unaware that learning derivatives fundamentally concerns understanding the rate of change [26]. Derivatives and the rate of change cannot be regarded as separate entities [6]. In high school syllabi, these concepts are often presented as distinct [12]. In Malaysian textbooks, the phrase 'rate of change' appears only in the application section of differentiation, positioned at the end of the chapter. Ironically, students need to develop an early conceptualization of derivatives as representing a rate of change [20]. However, Malaysian textbooks tend to emphasize derivatives as operators of functions, only later addressing applications. As a result, students may perceive derivatives merely as the outcomes of function differentiation or as algebraic processes involving derivative rules [27]. Here, derivatives are conveyed through an axiomatic approach, emphasizing procedural manipulation over conceptual understanding of differentiation as a process measuring change [19,28]. This pedagogical approach may cause students to associate derivatives predominantly with computational skills [3]. In response, Weber *et al*., [26] proposed an instructional model, grounded in calculus education research, advocating for the introduction of derivatives with an emphasis on the rate of change as the central concept. It is argued that such an approach reciprocally strengthens students' understanding of derivatives [19,26,28].

Given the foundational role of calculus in mathematics education, it is essential for teachers to possess a robust understanding of high school calculus content knowledge [8]. According to Ball *et al*.'s [2] Mathematical Knowledge for Teaching (MKT) framework, a teacher's Subject Matter Knowledge (SMK) forms the core mathematical foundation necessary for effective instruction. SMK encompasses the depth and coherence of conceptual knowledge required to accurately represent and explain mathematical ideas. Effective instruction in calculus, therefore, necessitates this deep and coherent subject matter understanding. Consequently, a teacher's concept image of the derivative [20] must be coherent and align with its formal mathematical definition to form a sound basis for teaching. Therefore, this study aims to investigate pre-service mathematics teachers' conceptual knowledge of derivatives, as they represent the next generation of educators responsible for fostering accurate and coherent concept images among their students. By investigating their knowledge, this research seeks to identify potential gaps or misconceptions that could hinder their future instructional effectiveness.

2. Statement of the Problem

Literature has documented numerous studies on derivatives and rate of change among teachers, university students, and high school students. A common issue that arises is an overemphasis on procedural aspects while neglecting the conceptual part [3,10,14,16]. It has been noted that this kind of teaching is more convenient and less time-consuming for both students and teachers [3]. However, building calculus knowledge solely on procedural understanding can lead to superficial learning, as it produces only short-term retention and may eventually discourage students from pursuing calculus-related fields [3]. Moreover, the National Council of Teachers of Mathematics (NCTM) in 2014 asserted that procedural knowledge is gained after the development of conceptual knowledge [16], indicating that conceptual knowledge should be prioritized in the teaching and learning process. However, findings from studies have shown that students often focus on differentiating expressions rather than interpreting their meaning [23]. Although some students can recognize rate of change in context, they explain it without connecting it to its deeper meaning [13]. For example, when finding turning points, students tend to focus on the steps of the differentiation process without understanding the role of derivatives in optimization [12,23]. They are accustomed to memorizing steps rather than understanding the underlying concepts. Conceptualizing derivatives in finding turning points should be emphasized to help students understand the use of rate of change in optimization [12].

Therefore, a basic understanding of calculus needs to be imparted to students through the calculus learning content provided in schools. Past researchers have emphasized the importance of understanding this concept, which is first introduced in school and falls under the teacher's responsibility [1,24]. Students' long-term learning will be adversely affected if teachers limit the derivative concept by persistently focusing on procedural approaches. Unfortunately, studies focusing on in-service teachers' knowledge of derivatives have revealed that teachers often resort to procedural methods when handling tasks [14] and admit that they are not entirely confident or clear about the derivative concept [7]. In fact, they have demonstrated a weak understanding of rate of change concept [6]. Given the importance of this issue, attention should also be directed toward mathematics teacher education programs, considering these unfavourable findings. Therefore, it is reasonable to conduct a similar study among pre-service mathematics teachers to investigate their knowledge of rate of change. Indeed, a local study revealed that Malaysian pre-service mathematics teachers performed poorly in the Teacher Education and Development Study in Mathematics (TEDS-M), with 57.1% demonstrating lower content knowledge and only 6.9% achieving a higher level [15]. Additionally, their mean scores were lower than the international mean [15]. Therefore, a qualitative research methodology was employed to thoroughly investigate Malaysian pre-service mathematics teachers' conceptual knowledge of rate of change, focusing on derivatives as an instantaneous rate of change. The research question was derived from the literature and the identified issue concerning rate of change: What kinds of knowledge of derivatives do Malaysian pre-service mathematics teachers have?

3. Methodology

Since this study aimed to gain an in-depth understanding of pre-service mathematics teachers' conceptual knowledge, the study employed an interpretivist paradigm and adopted a qualitative case study approach. A purposive sampling strategy was used to select three participants who could provide rich, information-laden cases relevant to the research focus [18]. The study was deliberately delimited to pre-service mathematics teachers at a public university in Malaysia to allow for a focused, contextual exploration. The primary selection criteria were twofold:

1. Academic Background: All participants were final-year mathematics education majors enrolled in a Bachelor of Education program, ensuring they had completed the required calculus and pedagogy courses relevant to the study.
2. Variation in Calculus Performance: Participants were purposively chosen from a moderate to high range of achievement in calculus to ensure the data captured a spectrum of understanding. The inclusion of high achievers was strategic, as their demonstrated procedural competency was expected to yield rich insights into the depth and nature of their

conceptual grasp of derivatives.

Three participants were deemed sufficient for an in-depth, qualitative case study as the goal was analytic depth rather than statistical generalisation [9]. A smaller sample allowed for detailed, task-based interviews and thorough analysis of each case, aligning with the study's exploratory and interpretive aims.The participants, referred to by the pseudonyms Misya, Amitha, and Zheng, were selected as follows:

1. Misya (Mathematics major, minor in Physical Education) earned an A in Calculus and an A in Pre-Calculus, with a CGPA of 3.74. She represents a consistently high achiever.
2. Amitha (Mathematics major, minor in English Language) earned a B in Calculus and an A– in Pre-Calculus, with a CGPA of 3.48. She represents a student with very good but not uniform high achievement.
3. Zheng (Mathematics major, minor in Chinese Language) earned an A– in Calculus and an A in Pre-Calculus, with a CGPA of 3.86. He represents another high achiever, but with a slightly different grade profile in calculus itself.

This deliberate variation in course grades was intended to explore whether and how different levels of prior calculus success might relate to conceptual understanding of derivatives as a rate of change.

A clinical task-based interview technique was used to directly analyze participants' knowledge, allowing for an in-depth investigation. This technique provides a comprehensive understanding of their knowledge, rather than focusing solely on correct or incorrect answers produced in paper-and-pencil tests, which often limit the investigation of knowledge and fail to reflect an individual's actual understanding [9]. All interviews were recorded through video recordings, and the data, including participants' written sheets and the researcher's field notes, were kept confidential.

Four tasks with different representations were used in this study, as detailed in the Appendix. Supplementary questions were asked based on participants' answers and responses to gain a more precise understanding of their knowledge. A panel of experts validated the instruments' content validity, rating them on a scale of three to five points. Tasks that scored three points were rephrased to improve clarity based on feedback from the panel. The instruments were adapted from previous research [19,25] and modified as needed to align with the objectives of this study.

Interview data were analyzed using the content analysis technique, specifically directed content analysis. The content of the interviews, field notes, and documents was analyzed to extract insights and nuances that deepen the understanding of the studied topic [18]. This process involves coding raw data and simultaneously developing categories to identify key information.

3. Results

Case 1: Misya

In Task 1 (See appendix), Misya stated that -6.45 is the result of differentiation, representing the slope of the function $f(x)$. She added that slope is the same as the rate of change, explaining, "It is the same as the rate of change because rate of change is one of the applications of derivatives." When asked why derivatives are related to the rate of change, she responded, "Hmmm, I'm not really sure why, but I've read that the result of differentiating a function gives the rate of change for that function. For example, $f'(x)$ gives the rate of change of $f(x)$. That's what I read, but I don't know the details."

In Task 2 (see Appendix), Misya initially described $\frac{dy}{dx}$ as differentiation with respect to x but later corrected herself, stating it represents the differentiation of y with respect to x. When asked to clarify what differentiation of y with respect to x means, she replied, "Function y, where x is the unknown in that function, and $\frac{dy}{dx}$ is the differentiation of that function with respect to x." Misya also referred to $\frac{\delta y}{\delta x}$ as a rate of change. When questioned about the difference between $\frac{\delta y}{\delta x}$ and $\frac{dy}{dx}$, she hesitated and said they are the same. She then stated that the relationship between $\frac{\delta y}{\delta x}$ and $\frac{dy}{dx}$ cannot be defined but later changed her answer, saying, "I think there is a relation, but I'm not sure."

In Task 3 (see Appendix), Misya explained, "To find rate of change, we usually use the difference in y divided by the difference in x." She provided an example from the table, "For instance, I take x values of one and two, and for $g(x)$, I take four and eight. So, the difference in $g(x)$ is eight minus four, divided by two minus one." She claimed that this formula could be applied to find the rate of change for the non-linear function $g(x)$.

In Task 4 (see Appendix), Misya hesitantly identified point *Q* as the correct answer, explaining that h represents the distance between points *P* and *Q*. She said, "I think it's at point *Q* because h is the distance from *P* to *Q*, but I'm not really sure, hmmm." She did not elaborate further due to her uncertainty. When asked about the first principles formula, Misya stated that it cannot be used to measure the rate of change because the limit causes h becomes zero. She explained, "Because of the limit, h becomes zero, so it becomes y_2 minus y_1, which gives a value. Then, in the denominator, if h is zero, dividing by zero would result in infinity." This indicates her misunderstanding that the denominator becomes zero, which she thought would lead leading to $\frac{dy}{dx}$ being infinity. When asked about her knowledge of the rate of change, she said, "For the rate of change, I think it must give a value. If we use the given formula (first principles), it will result in infinity."

In summary, Misya viewed derivatives as the slope of a graph and equated this with the rate of change, identifying it as an application of differentiation. However, she could not explain the connection beyond stating that differentiation yields a rate of change. This likely contributed to her incorrect selection of point *Q* in the first principles task. Misya failed to conceptualize derivatives as instantaneous rates of change, mistakenly asserting that the first principles formula cannot be used because the limit $h \to 0$ makes the expression undefined. She did not grasp that this limit represents the slope of a tangent line derived from secant lines approaching a point. Unsurprisingly, she also could not distinguish between $\frac{\delta y}{\delta x}$ and $\frac{dy}{dx}$, claiming both have the same meaning. Additionally, she applied the rate of change formula $\frac{y_2-y_1}{x_2-x_1}$ to the non-linear function $g(x)$, unaware that determining the rate of change in such cases requires infinitely small intervals to approximate the instantaneous rate.

Case 2: Amitha

In Task 1, Amitha identified -6.45 as the change in y over x. When asked if she knew anything else, she explained the differentiation process used to obtain -6.45, adding, "You get -6.45, which is the gradient, the rate of change."

In Task 2, Amitha described $\frac{dy}{dx}$ as the difference in y over x and the rate of change of y over x. She also stated that $\frac{\delta y}{\delta x}$ has the same meaning as $\frac{dy}{dx}$, explaining, "Because, as I said before, at our school, delta (δ) is written as d, so it's easier for us to write." Since Amitha viewed $\frac{dy}{dx}$ and $\frac{\delta y}{\delta x}$ as symbols with the same meaning, she was unable to define the relationship between the two.

In Task 3, Amitha explained that the rate of change could be obtained from the table because the given points could be plotted to create a graph, which would then provide a function to differentiate. When asked if the rate of change could be determined solely based on the table, she replied, “It can, but it’s difficult.” When asked to clarify the steps to find the rate of change after plotting the points, she simply answered, “Gradient.”

In Task 4, Amitha initially identified both points *P* and *Q* but later corrected herself, stating that point *P* is where the derivative is measured using first principles. She explained, “Because point *P* seems to be a turning or curve point… it seems at *P*, you get a more precise derivative.” In the next question, she stated that the first principles formula can be used to measure the rate of change, explaining, “Because it shows the difference in y and difference in x, so after you differentiate, you get the measurement for the rate of change.”

In summary, Amitha interpreted the derivative −6.45 as both a gradient and a rate of change, describing it as the change in y over x through differentiation. However, she did not identify it as an instantaneous rate of change. She viewed the first principles formula as similar to the rate of change formula, citing its reliance on differences in y and x, without recognizing its role in deriving the slope of a tangent line. Although she correctly identified point *P* as the point used in first principles, her reasoning that it is a turning point and thus more precise was superficial and lacked reference to secant lines converging on a tangent. This reflects a conceptual gap, further evident in her conflation of $\frac{dy}{dx}$ with $\frac{\delta y}{\delta x}$. Nevertheless, Amitha understood that the rate of change for a non-linear function $g(x)$ cannot be determined from a table alone. She noted that plotting the graph or differentiating the function is necessary, indicating some awareness that instantaneous rates require more than discrete values.

Case 3: Zheng

In Task 1, Zheng answered, “It is the rate of change reduced by 6.45.” He explained that it shows a reduction because of the negative sign. When he was asked if he knew anything else besides the rate of change, he answered, “Nothing.”

In Task 2, Zheng answered that $\frac{dy}{dx}$ represents the change of y with respect to x, and $\frac{\delta y}{\delta x}$ represents a small change for the rate of change. He said that there is a difference between the symbols because $\frac{\delta y}{\delta x}$ represents a very small change, and it is a smaller change compared to the change represented by $\frac{dy}{dx}$. He also mentioned that there is a relationship between $\frac{dy}{dx}$ and $\frac{\delta y}{\delta x}$, and it can be obtained through calculation, but he could not recall the details. He added, “If we want to find the rate of change, we can get it through $\frac{\delta y}{\delta x}$. It is also a kind of rate of change, but I forget in what context. $\frac{\delta y}{\delta x}$ can definitely represent $\frac{dy}{dx}$, but I already forgot.”

In Task 3, Zheng said that the rate of change is unobtainable from the given table and experienced difficulty explaining the reason. He said, “It looks like it has one, but… it’s difficult to say. I cannot find it because there is no function provided for me.” When he was asked what he understood about the term rate of change, he replied, “A change… hmmm, I do not remember how it changes. I think I do not really understand it in this context.”

In Task 4, Zheng answered that both points *P* and *Q* are used to measure the derivative using first principles. He explained, “Because of the limit, point *P* is the initial point, and point *Q* is the final point. We need to consider both *P* and *Q* and the domain in that.” He seemed to explain that points *P* and *Q* need to be considered in the derivative measurement. In the next question, he answered that the first principles formula can be used to measure the rate of change. He explained, "y_2 subtract y_1 gives the change, and then dividing by h, where h is like a range… so, when we divide like that, we can supposedly see how much it changes or how much the decrement is."

In summary, Zheng described −6.45 as a decreasing rate of change but did not identify it as an instantaneous rate. He viewed the first principles formula as a method for calculating the average rate of change, equating it with the difference in y-values divided by h, without recognizing the role of the limit in deriving a tangent slope. This misunderstanding led him to believe that both points *P* and *Q* are required for the calculation, reflecting a lack of understanding of how secant lines converge to a tangent line at point *P.* Zheng also claimed that $\frac{\delta y}{\delta x}$ represents a smaller rate of change than $\frac{dy}{dx}$, revealing further confusion about derivative notation and limits. While he correctly noted that the rate of change for a non-linear function $g(x)$ cannot be obtained from tabulated values, he failed to explain that such rates require approaching the point of interest with infinitely small intervals. Instead, he attributed the limitation to the absence of an explicit function and expressed difficulty understanding the task, indicating a shallow engagement with the underlying mathematical ideas.

Table 1

Comparison of conceptual knowledge across tasks for three cases

Task	Misya	Amitha	Zheng
Task 1: Interpreting *f'(x) = -6.45*	Result of differentiation, slope of the graph, and equated to rate of change.	Change in *y* over *x,* gradient, and rate of change.	A decreasing rate of change (due to negative sign).
Task 2: Meaning of *dy/dx* & *δy/δx*	Both represent "rate of change." Could not differentiate or define their relationship.	Both symbols mean the same thing (δ is written as *d*).	*dy/dx* = change of *y* wrt *x* while *δy/δx* = a small change. Claimed *δy/δx* is a **smaller** rate of change than *dy/dx*.
Task 3: Rate of Change from a Table (Non-linear *g(x)*)	Applied formula $(y_2-y_1)/(x_2-x_1)$ directly. Assumed the formula rate of change is applicable to the function ***g(x).***	Rate could be found by plotting points to get a function, then differentiating. Acknowledged difficulty from table alone.	Not obtainable because "there is no function provided." Struggled to explain conceptually.
Task 4 (a): Point of Derivative in First Principles	Selected Point Q (incorrect). Interpreted *h* as the distance between P and Q.	Selected Point P (correct) but provided inadequate reasoning.	Selected both Points P and Q (incorrect), seeing them as initial and final points for the interval *h*.
Task 4 (b): Understanding the Limit in First Principles	Formula is unusable because $h \to 0$ makes denominator zero, leading to undefined/infinity.	Formula is usable because it shows difference in *y* and *x*, yielding a rate of change.	Formula is usable as $(y_2-y_1)/h$ shows "how much it changes." Viewed it as an rate of change formula.
Overall Conceptual Knowledge	Procedural-superficial as she views derivative as an algebraic operator producing a "rate," but lacks connection to limit, tangent, or instantaneous rate. Confuses average and instantaneous.	Procedural with applied awareness as she links derivative to gradient and change, but explanations are procedural. Has developing awareness that instantaneous rate requires a function/continuum.	Fragmented & fuzzy as he has isolated ideas about "small change" but fails to synthesize them. Views first principles as an average rate formula. Knowledge is incoherent.

4. Discussion

Based on the findings, it can be concluded that all three pre-service teachers demonstrated either insufficient knowledge or misconceptions about derivatives. All three were unable to elaborate on how first principles work, which is essential to comprehend the slope of a tangent line at a point and the concept of instantaneous rate of change. Their explanations of derivatives were limited to describing them as a rate of change or the slope of a function, without further elaboration, highlighting their limited knowledge on the topic. Therefore, it was understandable that all of them provided either incorrect or insufficient explanations of the meaning and relationship between $\frac{dy}{dx}$ and $\frac{\delta y}{\delta x}$. In this context, the first principles formula leads to the visualization of imaginary secant lines approaching a tangent line at a point, thereby providing the instantaneous rate of change as the slope of the tangent line. This leads to the definition of a derivative as an instantaneous rate of change, which differs from rate of change in linear cases. It also helps in understanding the difference between $\frac{dy}{dx}$ and $\frac{\delta y}{\delta x}$, and their relationship expressed as $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$. Hence, the insufficient knowledge of these pre-service mathematics teachers should be further examined by analyzing and discussing their response patterns.

In Task 1, all three participants described derivatives as a rate of change, with Misya and Amitha further identifying them as the slope of a function. However, none elaborated on derivatives as the instantaneous rate of change or as the slope of a tangent line at a point. Their responses reflected a view of derivatives as mere computational tools rather than conceptual constructs. This suggests they comprehend $\frac{dy}{dx}$ as a numerical operator, focusing on procedures while overlooking its conceptual functionality. This lack of multiplicative reasoning between changing quantities likely shaped by their schooling [5]. In the Malaysian high school syllabus, derivatives are introduced as computational rules, with little emphasis on conceptual meaning. Although "gradient" is mentioned, its connection to instantaneous rate of change is not made explicit, potentially causing students to treat $\frac{dy}{dx}$ as a meaningless symbol. This issue has been highlighted by Kertil [12], who found that the school syllabus prioritizes algebraic manipulation over conceptual understanding, thereby limiting students' opportunities to grasp the practical significance of derivatives. Other studies [7,14,19,22,28] have also demonstrated that approaching derivatives through algebraic manipulation can hinder students' ability to appreciate the importance of calculus in real-world applications.

Unsurprisingly, none of the pre-service teachers demonstrated a proper understanding of first principles in Task 4, which involves deriving the slope of a tangent line and the instantaneous rate of change at a specific point. Although Amitha identified the correct point, her explanation was flawed, describing it as a turning point. The other two selected incorrect points, highlighting their conceptual confusion. None could visualize how secant slopes converge to the tangent slope, a key aspect of instantaneous rate of change. Zheng and Amitha viewed the formula as analogous to the general rate of change without recognizing the dynamic role of limits. Misya, meanwhile, misunderstood limits entirely, believing the formula was undefined due to division by zero. These misconceptions reflect a failure to grasp the derivative as a dynamic process. As Weber *et al*., [26] note, superficial understandings of limits can cause learners to treat derivatives as static pointwise operators, undermining their ability to visualize rate of change as continuous and dynamic, particularly the coordination of changing values in both x and h.

Consequently, in Task 2, all three participants were unable to accurately explain the meaning and relationship between $\frac{dy}{dx}$ and $\frac{\delta y}{\delta x}$. This is not surprising, as none of them demonstrated a clear cognition of first principles, which is essential for comprehending $\frac{dy}{dx}$and how it differs from $\frac{\delta y}{\delta x}$. This lacking may explain why Amitha claimed that both symbols represent the same meaning and purpose, reflecting her limited knowledge of $\frac{dy}{dx}$. Similarly, Misya stated that both symbols represent a rate of change but did not clarify whether they are interchangeable. On the other hand, Zheng displayed a misconception by claiming that $\frac{\delta y}{\delta x}$ represents a smaller rate of change compared to $\frac{dy}{dx}$. The participants' limited knowledge of derivative symbols reflects their insufficient conceptual grasp of derivatives, rooted in their inability to comprehend the first principles formula. This formula, which offers meaningful insight through graphical representation, is often neglected in favour of algebraic approaches preferred by both students and teachers [16]. This preference, especially during the introductory phase, warrants reconsideration. This study highlights the importance of symbolic derivatives, as they are introduced early in the upper secondary curriculum alongside their interrelated meanings. Thus, it is imperative that pre-service teachers acquire at least this foundational knowledge to teach derivatives effectively.

Task 3, which used a tabular representation, aimed to clarify participants' knowledge of derivatives as instantaneous rates of change. While Zheng and Amitha correctly noted that the rate of change for a non-linear function $g(x)$ cannot be directly obtained from discrete values, they failed to articulate the need for an infinite approach to determine it. Both claimed that an equation is necessary to compute the derivative, with Amitha adding that plotting the data to find the gradient could also work, reflecting her association of slope with rate of change. In contrast, Misya misunderstood the difference between linear and non-linear rates of change, using a simple rate of change formula on $g(x)$, and showing no awareness of variation in slope across points. This indicates her inability to distinguish between constant and instantaneous rate of change. Although "gradient" is often linked to first principles, its late introduction as a rate of change concept may confuse students, who perceive it as disconnected from earlier learning [21]. In Malaysia, gradient is introduced in lower secondary via algebraic and physical contexts, such as slope or road incline. Presenting it early as a representation of constant rate of change would better support students' later understanding of instantaneous rate of change in calculus [6,21,25].

In summary, the findings of this study revealed that pre-service mathematics teachers' knowledge of derivatives was insufficient and often incorrect. This was evident across the tasks, as they consistently provided inadequate explanations or incorrect statements about derivatives. The tasks were designed to cover fundamental derivative concepts introduced at the secondary level. However, the results were unfavourable, raising concerns about the depth of knowledge these pre-service teachers possess. This raises a critical question: If pre-service teachers lack a solid subject matter knowledge of derivatives, how can they effectively teach and develop pupils' understanding of the topic? Therefore, it is essential for instructors and lecturers in teacher training programs to adopt better approaches or modules that emphasize conceptual understanding alongside procedural skills. A balanced emphasis on both conceptual and procedural knowledge is essential, given the widespread application of derivatives in fields such as engineering, economics, business, biology, and other critical disciplines. Only by cultivating a deep, long-term understanding can pre-service teachers be sufficiently equipped to teach this fundamental topic effectively.

5. Conclusion and Recommendations

In conclusion, it is recommended that mathematics instructors and lecturers adopt technology-enhanced learning approaches to teach the concept of derivatives through first principles, as this method relies heavily on visualizing multiple secant lines. Integrating such technological tools can help pre-service teachers grasp the concept of limits and instantaneous change more effectively. Therefore, mathematics teacher education programs should prioritize the development of conceptual understanding alongside procedural fluency. Instructors should critically review and revise their Calculus and Pre-calculus modules to ensure they adequately address the “why” behind the rules, recognizing that their pedagogical choices directly shape the next generation of mathematics educators.

Furthermore, this study explored the derivative conceptual knowledge of a small, purposive sample of pre-service teachers. To build upon these findings, future research should involve a larger and more demographically representative sample of pre-service teachers, with the sample size determined by the population of the teacher education program to enhance the generalizability of the results. Extending this investigation to in-service teachers and high school students would also help determine if similar patterns of misunderstanding persist across different educational stages. Such research would provide a broader, systemic perspective on the challenges in calculus education, enabling more targeted interventions. Additionally, future studies could investigate pre-service teachers’ pedagogical content knowledge (PCK) in action, such as through the analysis of microteaching sessions on derivatives, to offer practical insights into bridging the gap between their own understanding and their future instructional practices.

References

[1] Ayebo, Abraham, Sarah Ukkelberg, and Charles Assuah. "Success in Introductory Calculus: The Role of High School and Pre-Calculus Preparation." *International Journal of Research in Education and Science* 3, no. 1 (2017): 11-19. https://doi.org/10.21890/ijres.267359

[2] Ball, Deborah Loewenberg, Mark Hoover Thames, and Geoffrey Phelps. "Content knowledge for teaching: What makes it special?." (2008).

[3] Borji, Vahid, Farzad Radmehr, and Vicenç Font. "The impact of procedural and conceptual teaching on students' mathematical performance over time." *International Journal of Mathematical Education in Science and Technology* 52, no. 3 (2021): 404-426. https://doi.org/10.1080/0020739X.2019.1688404

[4] Brijlall, Deonarain, and Zanele Ndlovu. "High school learners' mental construction during solving optimisation problems in Calculus: a South African case study." *South African Journal of Education* 33, no. 2 (2013): 1-18. https://doi.org/10.15700/saje.v33n2a679

[5] Byerley, Cameron. "Calculus students’ fraction and measure schemes and implications for teaching rate of change functions conceptually." *The Journal of Mathematical Behavior* 55 (2019): 100694. https://doi.org/10.1016/j.jmathb.2019.100694

[6] Byerley, Cameron, and Patrick W. Thompson. "Secondary mathematics teachers’ meanings for measure, slope, and rate of change." *The Journal of Mathematical Behavior* 48 (2017): 168-193.https://doi.org/10.1016/j.jmathb.2017.09.003

[7] Desfitri, Rita. "In-service teachers’ understanding on the concept of limits and derivatives and the way they deliver the concepts to their high school students." In *Journal of Physics: Conference Series*, vol. 693, no. 1, p. 012016. IOP Publishing, 2016. https://doi.org/10.1088/1742-6596/693/1/012016

[8] Vargas González, María Fernanda, José Antonio Fernández-Plaza, and Juan Francisco Ruiz Hidalgo. "Pre-service teachers’ understanding of the derivative of a function at a point." *International journal of mathematical education in science and technology* 54, no. 4 (2023): 483-510..

[9] Goldin, Gerald A. "A scientific perspective on structured, task-based interviews in mathematics education research." In *Handbook of research design in mathematics and science education*, pp. 517-545. Routledge, 2012.

[10] Hashemi, Nourooz, Mohd Salleh Abu, Hamidreza Kashefi, Mahani Mokhtar, and Khadijeh Rahimi. "Designing learning strategy to improve undergraduate students’ problem solving in derivatives and integrals: A conceptual framework." *Eurasia Journal of Mathematics, Science and Technology Education* 11, no. 2 (2015): 227-238. https://doi.org/10.12973/eurasia.2015.1318a

[11] Herbert, Sandra, and Robyn Pierce. "Revealing educationally critical aspects of rate." *Educational Studies in Mathematics* 81, no. 1 (2012): 85-101. https://doi.org/10.1007/s10649-011-9368-4

[12] Kertil, Mahmut. "Conceptual analysis of derivative as a rate of change and analysis of the mathematics textbooks." *Sakarya University Journal of Education* 11, no. 3 (2021): 545-568.https://doi.org/10.19126/suje.977200

[13] Kertil, Mahmut, and Hande Gülbağcı Dede. "Promoting Prospective Mathematics Teachers' Understanding of Derivative across Different Real-life Contexts." *International Journal for Mathematics Teaching and Learning* 23, no. 1 (2022): 1-24.
[14] Lam, Toh Tin. "On in-service mathematics teachers' content knowledge of calculus and related concepts." *The Mathematics Educator* 12, no. 1 (2009): 69-86.
[15] Leong, Kwan Eu, Cheng Meng Chew, and Suzieleez Syrene Abdul Rahim. "Understanding Malaysian pre-service teachers mathematical content knowledge and pedagogical content knowledge." *Eurasia Journal of Mathematics, Science and Technology Education* 11, no. 2 (2015): 363-370.
[16] Makgakga, Sello, and Eva G. Maknakwa. "Exploring learners' difficulties in solving grade 12 differential calculus: A case study of one secondary school in Polokwane district." In *Proceedings: Towards Effective Teaching and Meaningful Learning in Mathematics, Science and Technology. ISTE International Conference on Mathematics, Science and Technology Education*, pp. 23-28. 2016.
[17] Marsitin, R. "Analysis of differential calculus in economics." In *Journal of Physics: Conference Series*, vol. 1381, no. 1, p. 012003. IOP Publishing, 2019. https://doi.org/10.1088/1742-6596/1381/1/012003
[18] Merriam, S. B. (2009). Qualitative research: A guide to design and implementation (2nd ed.). Jossey-Bass. https://doi.org/10.1097/NCI.0b013e3181edd9b1
[19] Orton, Anthony. "Students' understanding of differentiation." *Educational studies in mathematics* 14, no. 3 (1983): 235-250. https://doi.org/10.1007/BF00410540
[20] Tall, David, and Shlomo Vinner. "Concept image and concept definition in mathematics with particular reference to limits and continuity." *Educational studies in mathematics* 12, no. 2 (1981): 151-169.
[21] Teuscher, Dawn, and Robert E. Reys. "Rate of change: AP calculus students' understandings and misconceptions after completing different curricular paths." *School Science and Mathematics* 112, no. 6 (2012): 359-376. https://doi.org/10.1111/j.1949-8594.2012.00150.x
[22] Tyne, Jennifer G. "Calculus students' reasoning about slope and derivative as rates of change." (2016).
[23] Ubuz, Behiye. "Interpreting a graph and constructing its derivative graph: Stability and change in students' conceptions." *International Journal of Mathematical Education in Science and Technology* 38, no. 5 (2007): 609-637. https://doi.org/10.1080/00207390701359313
[24] Wade, Carol H., Gerhard Sonnert, Philip M. Sadler, and Zahra Hazari. "Instructional experiences that align with conceptual understanding in the transition from high school mathematics to college calculus." *American Secondary Education* (2017): 4-21.
[25] Weber, Eric, and Allison Dorko. "RETRACTED: Students' and experts' schemes for rate of change and its representations." (2014): 14-32.https://doi.org/10.1016/j.jmathb.2014.01.002
[26] Weber, E., M. Tallman, C. Byerley, and P. W. Thompson. "Introducing derivative via the calculus triangle." *Mathematics Teacher* 104, no. 4 (2012): 274-278.
[27] Weber, Eric, and Patrick W. Thompson. "Students' images of two-variable functions and their graphs." *Educational Studies in Mathematics* 87, no. 1 (2014): 67-85. https://doi.org/10.1007/s10649-014-9548-0
[28] Zandieh, Michelle. "A theoretical framework for analyzing student understanding of the concept of derivative." *CBMS issues in mathematics education* 8 (2000): 103-127.https://doi.org/10.1090/cbmath/008/06

Appendix

1. A function, $f(x)$ has a derivative of $f'(2.5) = -6.45$ at point $x = 2.5$. Hence, convey what does -6.45 meant?
2. $$\frac{dy}{dx} \text{ and } \frac{\delta y}{\delta x}$$
   The symbols of differentiation are given as shown above. Hence, describe the meaning for each of differentiation symbols below.
   a) $\frac{dy}{dx}$
   b) $\frac{\delta y}{\delta x}$
   c) Can you define the relationship between $\frac{dy}{dx}$ and $\frac{\delta y}{\delta x}$? How?
3. Table below shows the values of $g(x)$ and x for a non-linear function g.

Table 2: Non-linear function $g(x)$

x	$g(x)$
1	4.0

Journal of Advanced Journal of Advanced Research in Social and Behavioural Sciences
Volume 43, Issue 1 (2026) 193-204

2	8.0
3	10.0
4	6.5
5	4.5
6	15.0
7	9.0

Can you find rate of change for the function g? Why?

4. Graph in Figure 1 illustrated first principles formula shown below which represent definition for derivative.

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{y_2 - y_1}{h}$$

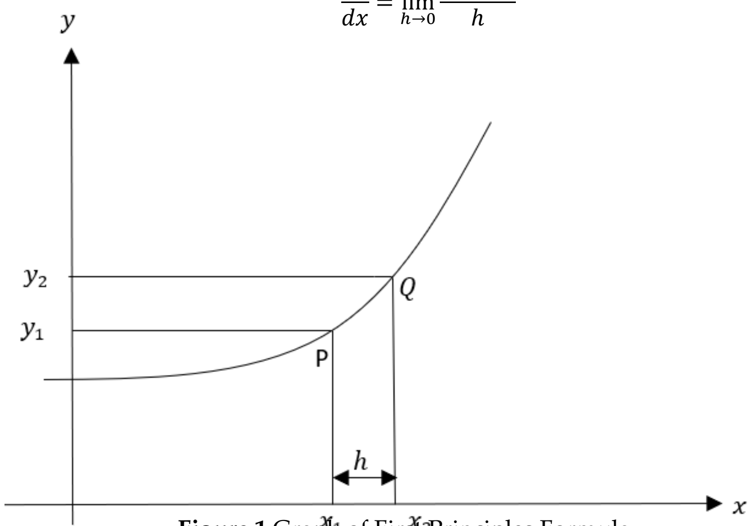

Figure 1 Graph of First Principles Formula

a) Which point(s) on the graph does measure derivative as stated by the formula above? Why?
b) Do you think the stated formula above can be defined as measurement for rate of change? Why?