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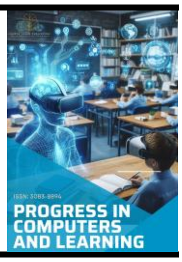

Redesign Kanji Character by using Bezier and Wang-Ball Curves

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ABSTRACT

Mathematical expressions can be used in computer-aided geometric design to construct surfaces and curves. This study depicted a 2D object's outline, such as a Kanji character. Kanji calligraphy suggests the Kanji character that represents actual objects. The art of calligraphy is distinct and can only be created by a skilled individual. Kanji calligraphy can be vectorized and saved in digital format to conserve the arts for future use. This study compares the degree elevation of the Bezier and Wang ball curves based on the Kanji calligraphy character "Hana" and uses the curves to rebuild "Hana." The original scanned image of the "Hana" character is compared to the degree elevation for both the Wang ball curve and the Bezier curve methods after Bernstein polynomials are used in the MATLAB software. The degree is assessed up to the fourth degree. The character's redesign will be based on the comparison's best degree. The time it takes to compute the curves is also noted. With a computation time of 0.2094 seconds, the result indicates that the degree four of the Bezier curve is the optimal curve to employ for redesigning the "Hana" character. Since degree four Bezier curves required the least amount of time to construct and demonstrate that Bezier curves are far simpler than Wang ball curves, it can be said that this degree is the greatest choice for redesigning the "Hana" character.

1. Introduction

1.1 Overview

The field of computer-aided geometric design (CAGD) is a subfield of computational geometry and computer graphics that focuses on representing geometric shapes using algorithms. Recent study by Wei *et al.,* [21] presents a new method for addressing uncertainty in surface approximation: the hesitant fuzzy Bézier surface (HFBS) approximation model, which blends hesitant fuzzy sets with geometric modeling. Ameer *et al.,* [3] cited that CAGD employs surfaces and curves as mathematical models to support the design process in industries like architecture, aerospace, and automobiles. In this current era, technology has a significant impact on art and design, Bawono *et al.,* [5] state this

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situation particularly in the manufacturing phase where computer-based technology software is commonly utilized in creating design. French engineer Pierre Bezier established CAGD in the mid-1960s, and its foundational ingredient is Bezier curves. During his time at Renault, Bezier created car body forms using these curves. A study by Eze *et al.,* [7] mentioned about how Bezier curve employed in computer graphics, modelling and producing smooth surfaces and curves for usage in animation and font creation. Study by Mahmoud *et al.,* [13], Bezier curves are so flexible, they can be used to approximate shapes with a limited number of control points, which is why CAGD frequently uses them. Karateke [12] study found that changing the control points allows for the flexible and unrestricted creation of shapes using Bezier curves. A paper by Arslan and Aron [4] state the interpolation properties and convex hull also enable the designers to produce easily adjustable and smooth curves.

Ramanantoanina *et al.,* [14] claim that by utilizing the correspondence between the rational Bezier and the interpolating barycentric form, as well as by examining the impact of modifying the latter's degrees of freedom (interpolation points, weights, and nodes), it is possible to achieve more direct control over the curve shape. To satisfy the demands of the manufacturing sector, curves and surfaces with flexible shapes and adjustable lengths and sizes are needed. Consequently, with this degree of flexibility and adaptability, researchers produce a large number of aesthetically pleasing Bezier curves and surfaces as mentioned in Said *et al.,* [1]. On the other hand, the Wang-Ball curves method is rarely employed in research and is often overlooked. Although less prevalent than Bezier curves in computer graphics, this kind of curve was utilized in Chinese calligraphy and several stylized writing systems. They can assist in simulating the contour and flow of brush strokes in Kanji characters and are mostly concerned with seamless transitions.

The Wang-Ball curves offer a versatile building block for geometric modelling by defining a parametric curve that extends the Bezier curves. Study by Zheng *et al.,* [22] state that the capacity to maintain specific geometric qualities, such as shape control and continuity, is what distinguishes the curves, which are defined using a collection of control points. Bhatti and Sharmila [6] explained that Wang-Ball curves are positioned for applications that require curve shape precision because they use a mathematical formulation of polynomial basis functions for localized shape control. Alaidroos [2] study found that controlling local forms is essential because Wang-Ball curves are often especially useful in surface interpolation and advanced geometric modelling. According to Hu and Bo [11], Wang-Ball curve enable greater design freedom by allowing for smoother transitions and more control with fewer points which making it highly helpful in CAGD for precisely modelling complicated surfaces with ease of manipulation.

The purpose of this study was to use the Wang-Ball and Bezier curves to generate the selected character, "Hana" 花, meaning flower. The meaning of the Kanji character, which includes flower, sun, and bird, served as the inspiration for this one. However, the intricacy of mastering the delicate and sophisticated writing techniques required to use ink and brush makes it extremely impossible for regular people to write visually pleasing calligraphy without a significant amount of practice. It might take decades for a calligrapher to become proficient in "shodo" and create their unique writing style. Research by Shams *et al.,* [17] mentioned that Japanese calligraphy, or Kanji characters, is more intricate and has many strokes than Arabic calligraphy.

As mentioned in research by Sharma [15], the quartic trigonometric Bezier models are still relatively new in development, which means that their full range of uses and effectiveness has yet to be fully explored or realized. The similar method is suggested in this study for vectorizing Kanji characters in calligraphy papers by employing the Bezier and Wang-Ball curves to connect each Kanji character stroke. It can be expressed in any higher degree to the quartic Bezier to explore the design of Kanji character. After that, the Wang-Ball and Bezier curves were compared. The segments of strokes with the highest accuracy and flexibility for shape creation are obtained from the best results between the degree elevation of the Bezier and Wang-Ball curves on Kanji characters. Therefore, by taking into account their assessed degrees, it will decide which curves are suitable for redesigning the "Hana" character.

2. Theory

2.1 Bezier Curves Method

Bezier curves were invented by a French engineer named Pierre Bezier close to the end of the 1950s between 1958 and 1960, for the Renault Auto Company modelled automobile body shapes. Paper by Mahmoud *et al.,* [19] showed that these curves were defined by Bezier using Bernstein polynomials and De Casteljau algorithm. The nature of these curves exhibited by the ability to transition smoothly from point to point as mentioned in Rababah and Moath [16].

Originally Bezier curves mentioned in Farin [8] as:

$$V(t)=\sum_{i=0}^{n} v_i B_i^n(t) \tag{1}$$

whereas,

$B_i^n(t)$ is a Bernstein's polynomials where $i = 1, 2, 3, \ldots$

v_i is the control points where $i = 0, 1, 2, 3, \ldots$

In this case, Bernstein's polynomials can be solved by

$$B_i^n(t)=\binom{n}{i} t^i (1-t)^{n-i} \tag{2}$$

whereas, $i = 0, 1, 2, 3, \ldots$ and n is the degree

2.2 Wang-Ball Curve Method

In order to create higher degree Ball curves, Wang [20] proposed Wang-Ball curves in 1987. These curves extend the polynomial degree of conventional Ball basis functions as in Hu *et al.,* [10]. In accordance with Hamza *et al.,* [9] and Hu *et al.,* [18], the Wang-Ball is defined for 0 < t ≤ 1 as follows in Eq. (3):

$$W(t)=\sum_{i=0}^{n} V_i W_i^n(\mathrm{t}) \quad , 0 \le t \le 1 \tag{3}$$

V_i is the control points where $i = 0, 1, 2, 3, \ldots$

$W_i^n(t)$ is the Wang-Ball basis functions for degree number *n*

$W_i^n(t)$ can be solved according to the number of degree *n.*

If the number of degree *n* is even, then the Wang-Ball basis function is:

Even: Degree 2, 4 $$W_i^n(t)=\begin{cases}(1-t)^2W^i & ,\ 0\le i\le \frac{n}{2}-1\\ (1-t)^1W^{\frac{n}{2}} & ,\ i=\frac{n}{2}\\ t^2W^{n-i} & ,\ \frac{n}{2}+1\le i\le 3\end{cases} \tag{4}$$

If the number of degree *n* is odd*,* the Wang-Ball basis function solution is as follow:

Odd: Degree 3, 5 $$W_i^n(t)=\begin{cases}(1-t)^2W^i & ,\ 0\le i\le \frac{n-3}{2}\\ (1-t)^1W^{\frac{n-1}{2}} & ,\ i=\frac{n-1}{2}\\ tW^{\frac{n-1}{2}} & ,\ i\le \frac{n+1}{2}\\ t^2W^{n-i} & ,\ \frac{n+3}{2}\le i\le 3\end{cases} \tag{5}$$

3. Methodology

The flowchart in Figure 1 below shows the procedures needed to redesign the Kanji character:

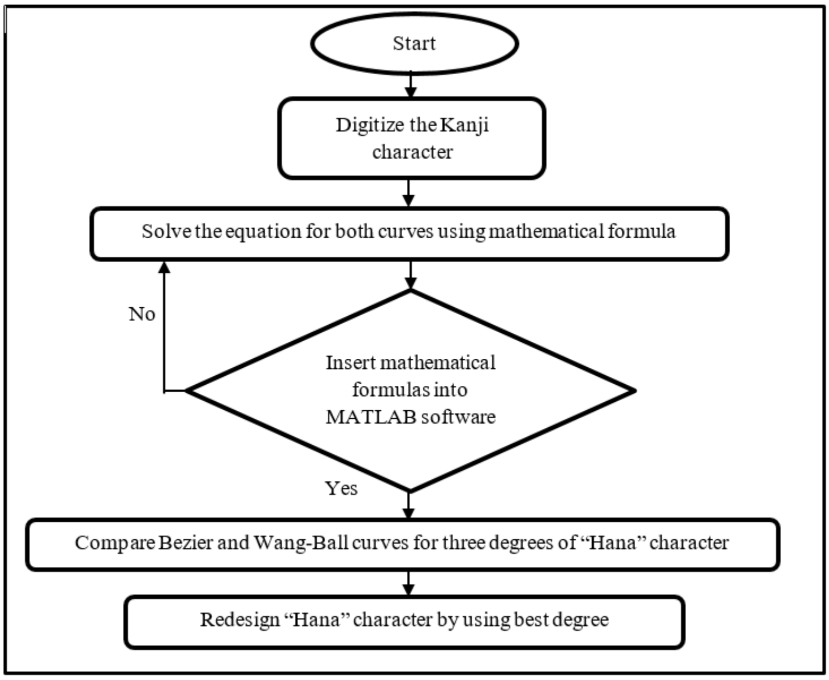


Fig. 1. Methodology flowchart

There are four steps involved in redesigning a Kanji character. Plot the points for each contour of the selected item using graph paper after first identifying it. The next step is to solve the equation for the Wang-ball and Bezier curves using a mathematical process. Next, check to see if the equations generate the correct curve by entering the solved mathematical formulas into the MATLAB application. If the equations yield the right curves, phase four will start, comparing the Bezier and Wang-ball curves for particular degrees. If the equations give contradicting execution, phase three needs to be repeated to confirm it. To ascertain which degree yields the best curve shot, phase four compares the Wang-ball and Bezier curves for the evaluated degree. The time needed to redesign the "Hana" character utilizing the best degree from the Wang-ball and Bezier curves will be noted after those four steps are finished.

3.1 Phase I: Digitize the Kanji Character

The character "Hana," which translates to "flower," was selected for this study. To obtain the points, an illustration of "Hana" was printed on graph paper. The Kanji calligraphy for "Hana" is displayed in Figure 2 above. In total, this character has seven strokes. Each stroke, though, has a unique curve shape.

Fig. 2. An illustration of the kanji character "Hana."

3.2 Phase II: Solve the Equation of Both Curves using Mathematical Formula

The second stage involves applying mathematical formulas to both approaches before utilizing MATLAB software to create the curves. The MATLAB software will be used to plot the curves for the Bezier and Wang-Ball curves using the final equation for each degree produced in this phase.

3.2.1 Bezier curve

a) Degree Two

The basis function from Eq. (1) and Eq. (2) was used to calculate for degree two. The degree two polynomial is provided below.

$$B_0^2 = \binom{2}{0} t^0 (1-t)^{2-0} = (1-t)^2 = t^2 - 2t + 1 \qquad (6)$$

$$B_1^2 = \binom{2}{1} t^1 (1-t)^{2-1} = 2t(1-t) = -2t^2 + 2t \quad (7)$$

$$B_2^2 = \binom{2}{2} t^2 (1-t)^{2-2} = t^2 \quad (8)$$

Then, from the result, summarize Eq. (6), Eq. (7) and Eq. (8) into the matrix form as follows:

$$V(t) = \begin{bmatrix} t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix}$$

$$V_2(\mathrm{t}) = (1-\mathrm{t})^2 v_0 + 2t(1-t)v_1 + t^2 v_2 \quad (9)$$

b) Degree Three

$$V_3(\mathrm{t}) = (1-\mathrm{t})^3 v_0 + 3t(1-t)^2 v_1 + 3t^2 v_2 + t^3 v_3 \quad (10)$$

c) Degree Four

$$V_4(\mathrm{t}) = (1-\mathrm{t})^4 v_0 + t(1-t)^3 v_1 + t^2(1-t)^2 v_2 + t^3(1-t)v_3 + t^4 v_4 \quad (11)$$

where v_0, v_1, v_2, v_3 *and* v_4 exist in the Eqs. (9), (10) and (11) are the control points.

3.2.2 Wang-Ball curve

The Wang-Ball curve is generated using two different basis functions in contrast to the Bezier curve because it takes into account the degree of the polynomial, which can be either odd or even.

Even degree

a) Degree Two

Since degrees two and four are even, the equation was derived using Eq. (4). The Wang-Ball curve's degree two derivation is as follows:

$$W_0^2(t) = (1-t)^2 = t^2 - 2t + 1 \quad (12)$$

$$W_1^2(t) = 2t(1-t)^1 = -2t^2 + 2t \quad (13)$$

$$W_2^2(t) = t^2 \quad (14)$$

Then, substitute Eq. (12), Eq. (13) and Eq. (14) into the matrix gives a result

$$\begin{bmatrix} t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix}$$

$$V_2(\mathrm{t}) = (1-\mathrm{t})^2 v_0 + 2t(1-t)v_1 + t^2 v_2 \tag{15}$$

b) Degree Four

$$V_4(\mathrm{t}) = (1-\mathrm{t})^2 v_0 + (-2t^4 + 6t^3 - 6t^2)v_1 + (4t^4 - 8t^3 + 4t^2)v_2 + (2t^3 - 2t^4)v_3 + t^2 v_4 \tag{16}$$

where v_0, v_1, v_2, v_3 and v_4 exist in the Eq. (15) and Eq. (16) are the control points.

Odd degree

a) Degree Three

For degree three, the Eq. (5) to formulate its basis function as follows:

$$W_0^3(t) = (1-t)^2 W^0 \tag{17}$$

$$W_1^3(t) = (1-t)^1 W^1 \tag{18}$$

$$W_2^3(t) = tW^1 \tag{19}$$

$$W_3^3(t) = t^2 W^0 \tag{20}$$

Then, same goes as previous degree, summarize the equations into matrix form as follows.

$$\begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & -2 & 0 \\ 2 & -2 & 2 & 1 \\ -2 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$V_3(\mathrm{t}) = (1-\mathrm{t})^2 v_0 + (1-t)(2\,\mathrm{t}(1-\mathrm{t}))v_1 + t(2t(1-t))v_2 + t^2 v_3 \tag{21}$$

3.3 Phase III: Insert the Mathematical Formulas into MATLAB Software

Equations are then entered into MATLAB software once they have been obtained for every degree. The control polygon comprises a set of verified, chosen control points is linked to both types of depicted curves. Carefully selecting preferred or best control polygons is necessary to ensure that they precisely fit the intended curve. After the codes are run in the software, the object's plotted graph is displayed. If the algorithms produce an image that differs from the original item, we must verify the control polygons or earlier equations. Next, verify and repeat the codes until the image resembles the thing.

3.4 Phase IV: Compare Bezier and Wang-Ball Curves for Three Degrees of "Hana" Character
3.4.1 Quadratic curve

A quadratic Bezier curve can be defined by a straightforward curve segment with three control points included. It is less flexible than curves with more degrees. They work well for some applications because they utilize less processing power. Their limited number of control points, however, limits the intricacy of the curves they can represent. The quadratic curve is generated using MATLAB, as seen in Figure 3 below.

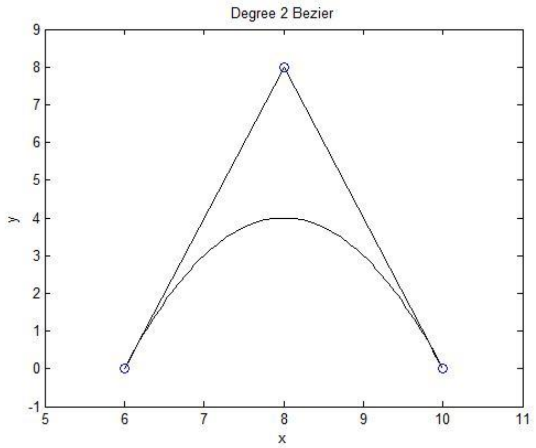
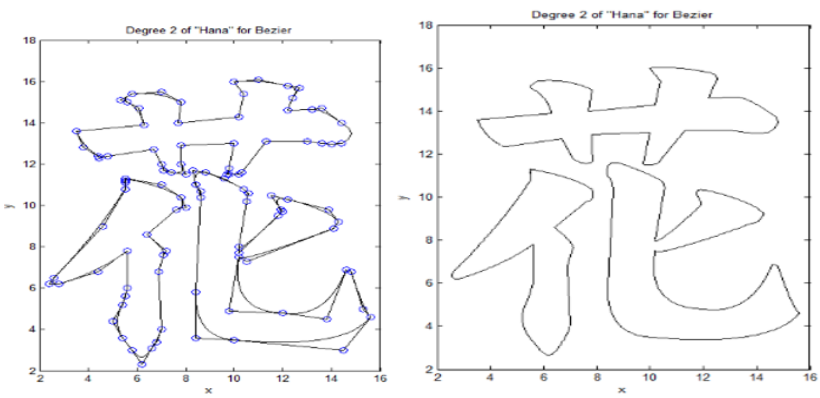


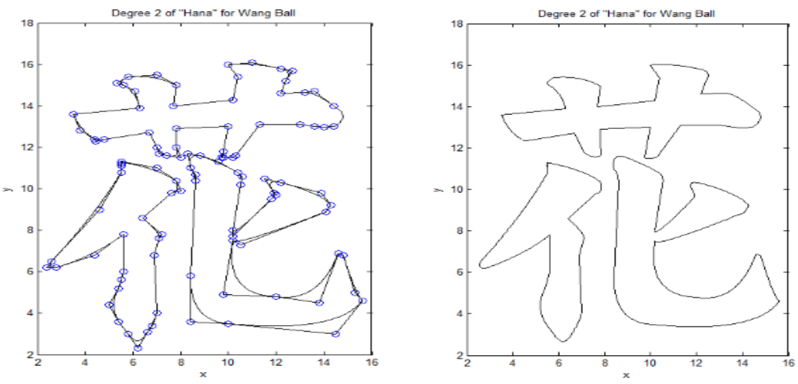
Fig. 3. Quadratic curve

To create the "Hana" character, the polynomial for both curves in Eq. (9) and Eq. (15) was inserted into the code. This character was generated using both linear and quadratic curves. The results of the Hana character using a Bezier curve are displayed in Figures 4 and 5 below, both with and without a control polygon.



(a) with control polygon (b) without control polygon

Fig. 4. "Hana" character by using quadratic Bezier curve with and without control polygon



(a) with control polygon (b) without control polygon

Fig. 5. "Hana" character by using quadratic Wang-Ball curve with and without control polygon

3.4.2 Cubic curve

An extra curve segment with four control points defines a cubic Bezier curve. Compared to curves with fewer degrees, it is more flexible. Simple curved shapes can be made with cubic Bezier curves. Because they don't require as much processing power, they are effective for some applications. However, the complexity of the curves they may represent is limited by their vast number of control points. The Cubic Curve's creation is depicted in Figure 6 below. Generation for cubic curve for both curves shown in Figure 7 and 8.

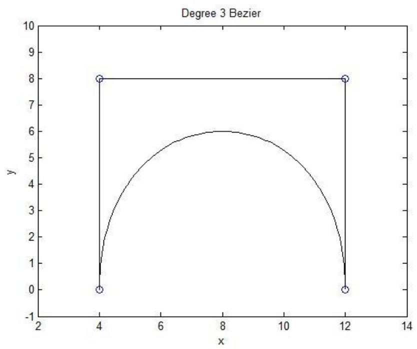
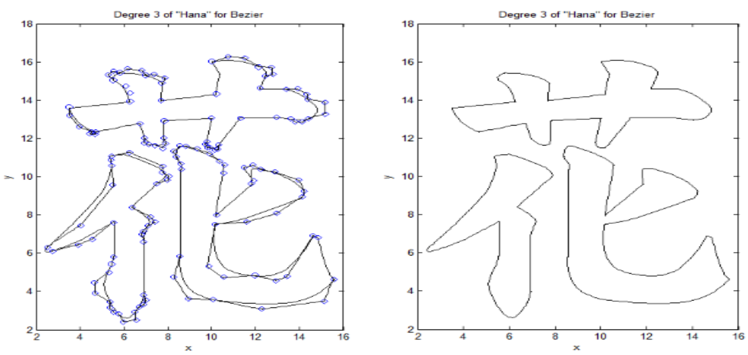
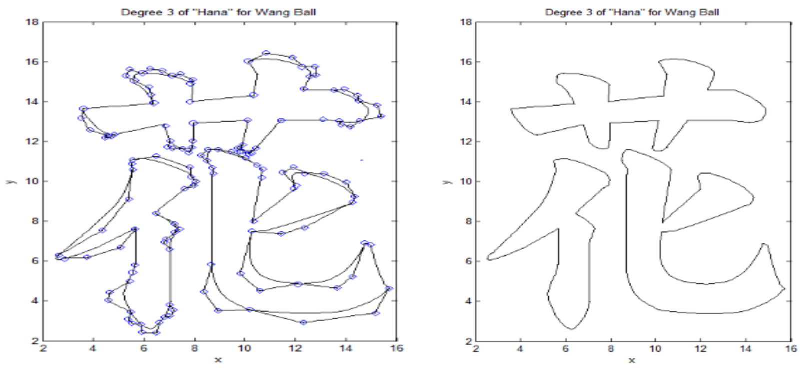


Fig. 6. Cubic Curve



(a) with control polygon (b) without control polygon

Fig. 7. "Hana" character by using cubic Bezier curve with and without control polygon



(a) with control polygon (b) without control polygon

Fig. 8. Hana" character by using cubic Wang-ball curve with and without control polygon

3.4.3 Quartic curve

The complexity of curve shaping increases when a quartic Bezier curve is extended by five control points. Compared to cubic Bezier curves, quadratic curves offer greater flexibility, allowing for the representation of intricate shapes. When the curve needs to be more accurately altered, they are used. Figure 9 below illustrates how to use MATLAB to generate the Quartic Curve. The "Hana" character was displayed in Figure 10 and 11 below.

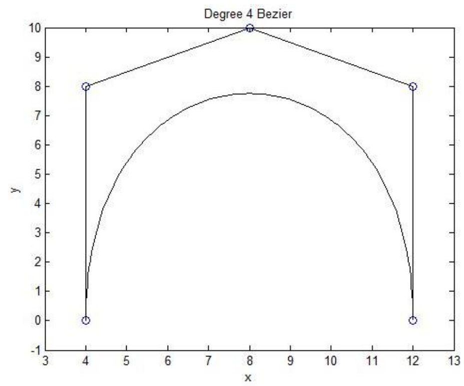
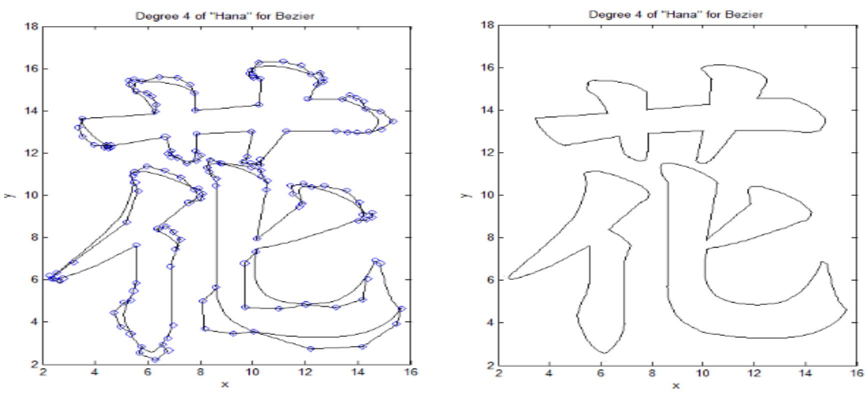
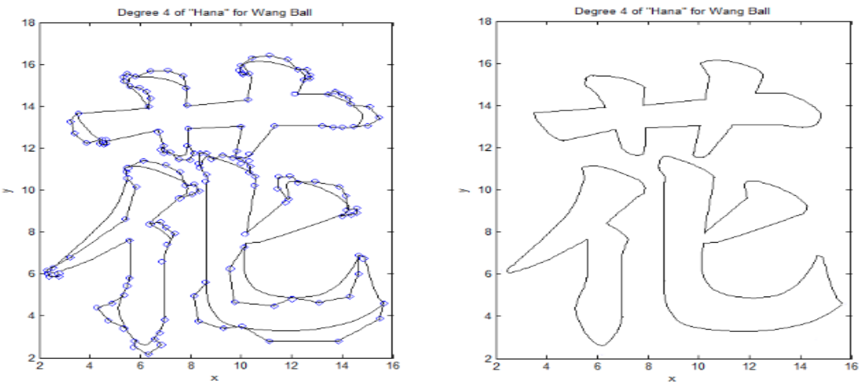


Fig. 9. Quartic curve



(a) with control polygon (b) without control polygon

Fig. 10. "Hana" character by using quartic Bezier curve with and without control polygon

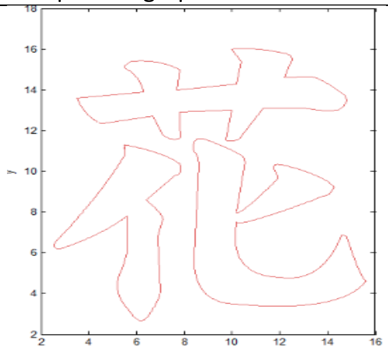
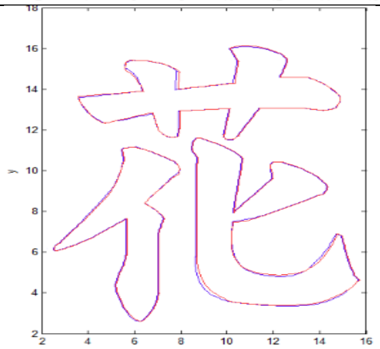
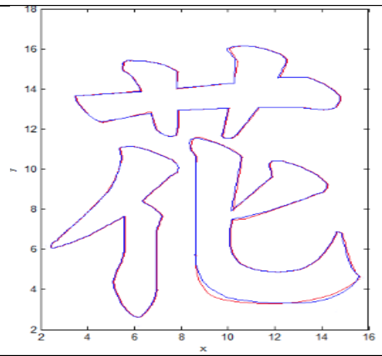


(a) with control polygon (b) without control polygon

Fig. 11. "Hana" character by using quartic Wang-ball curve with and without control polygon

4.1 Phase V: Compare Bezier and Wang-Ball curves for Selected Degrees of "Hana" Character

Table 1
Comparison curve of Bezier and Wang-ball curves

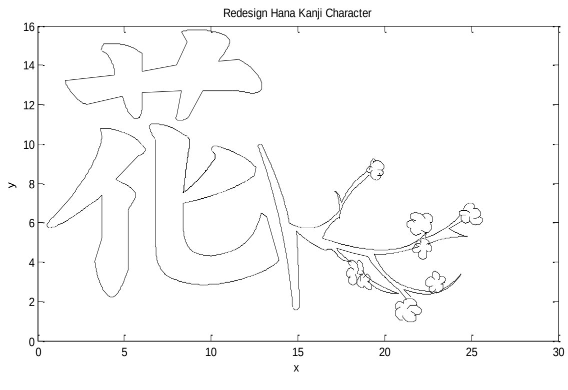
Degree	Comparison graph
Degree two	
Degree three	
Degree four	

This section compares the generation of the Kanji character "Hana" for the Bezier and Wang Ball curves. First, MATLAB software was used to generate each curve independently by entering each polynomial. Then, in order to compare the degrees, both curves were plotted on the same graph. The Wang-ball curve is depicted by the red outline, whereas the Bezier curve is indicated by the blue outline. Table 1 shows a Bezier and Wang-Ball curve with a degree of 2. Because the polynomial used in both curves produced the same result for degree 2, both curves yielded the same outcome. Eqs. (6), (7), (8), (12), (13) and (14) demonstrate this. Thus, it can be concluded that the shapes of the two degree 2 curves are comparable. For degrees three and four, the "Hana" character displays a different outcome at the curved region where the blue Bezier curve outline is more curved than the red Wang-Ball curve. Due to a change in the polynomial throughout the degree elevation, the curves had slightly different results. According to the comparative result, the Bezier curve's degree four provides a superior curve form than the Wang-ball.

4.2 Phase VI: Redesign “Hana” Character by using the Best Degree

The next procedure was to redesign the character “Hana”, where the plum blossom is added to reflect the blossoms in Japan adjacent the character. The plum blossom's stalks and blooms were constructed using a Bezier curve with degree four for the final image of the revised Hana kanji character. The addition of plum blossom enhances the character's beauty without altering the word's meaning. The outcome in Figure 12 demonstrates the difference between before and after redesign of “Hana” character. In this design redesign character produces computation time of 0.2094 seconds, thus conclude that the fourth degree of the Bezier curve is the optimal curve to employ for redesigning the "Hana" character.

a) Original **“**Hana” kanji character



b) redesign “Hana” kanji character

Fig. 12. Comparison results of original and after redesign “Hana’ kanji character

5. Conclusions

The "Hana" Kanji character was redesigned in this study utilizing Wang-ball and Bezier curves. Many studies have been conducted in the past, but they have not specifically addressed the distinction between Wang-ball and Bezier curves. This study's primary goal is to compare the degree elevation of the Wang ball and Bezier curves using the "Hana" character. In order to compare the Bezier and Wang ball curves, mathematical formulas for degrees two, three, and four are entered into MATLAB software. The curves are then contrasted with the object's original scanned image. Then, the computation time and the optimal degree of the Wang and Bezier ball curves were found. With a computation time of 0.2094 seconds, degree four of the Bezier curve is the optimum degree elevation. The second goal is to concentrate on redesigning the "Hana" character. The character was redesigned using the Bezier curve's degree four. The "Hana" character now includes parts of the plum blossom, which is one of Japan's most beautiful flowers.

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